

Trig Equations

(a) Express the equation

$$5 \sin 2x - 4 \cos 2x = 0$$

in the form

$$\tan 2x = k,$$

where k is a constant.

(2 marks)

(b) Hence find all solutions of the equation

$$5 \sin 2x - 4 \cos 2x = 0$$

in the interval $0^\circ < x < 180^\circ$, giving your answers to the nearest 0.1° .

(4 marks)

Question Number and part	Solution	Marks	Total	Comments
6(a)	Awareness that $\frac{\sin \theta}{\cos \theta} = \tan \theta$ $\tan 2x = 0.8$	M1 A1	2	Generous Correctly derived – not fudged
(b)	\tan^{-1} (candidate's k) $2x = 38.7^\circ$ $2x = 38.6598..^\circ \Rightarrow x = 19.3^\circ$ $2x = 218.6598..^\circ \Rightarrow x = 109.3^\circ$	M1 A1 A1✓ A1✓	4	$38.6598...^\circ$ 0.6747 rads condone 38.6° or better $19.3299...^\circ$ ft half their $\tan^{-1} k$ accept 0.337 rads Their previous value + 90° Must be degrees for final mark Lose final A1 for extra solutions in interval
Total			6	

(a) Given that

$$7 \sin^2 x - 2 \cos^2 x + 6 \sin x = 1,$$

show that

$$3 \sin^2 x + 2 \sin x - 1 = 0.$$

(2 marks)

(b) Hence find all solutions of the equation

$$7 \sin^2 x - 2 \cos^2 x + 6 \sin x = 1,$$

in the interval $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest 0.1° .

No credit will be given for an approximate numerical method.

(5 marks)

4 (a)	$\dots \Rightarrow 7\sin^2x - 2(1 - \sin^2x) + 6\sinx = 1$ $\Rightarrow 9\sin^2x + 6\sinx - 3 = 0$ $\Rightarrow 3\sin^2x + 2\sinx - 1 = 0.$	M1		Use of $\cos^2x = 1 - \sin^2x.$
(b)	$\dots \Rightarrow (\sinx + 1)(3\sinx - 1) = 0$ $\Rightarrow (\sinx) = -1 ; (\sinx) = \frac{1}{3}$ (allow 0.33)	A1	(2)	AG Full valid completion
	$x = 270^\circ;$ $x = 19.5^\circ;$	M1		Attempt to factorise or use the formula
	$x = 180 - 19.5 = 160.5^\circ;$	A1		Both values – accept unsimplified form from formula
		B1		
		A1		Deduct max one mark for any wrong answer in interval
		A1 ft	(5)	ft on $180 - "19.5"$
		TOTAL	(7)	

(a) Find all the solutions of the equation

$$\sin(3x + 45^\circ) = 0.7$$

in the interval $-90^\circ \leq x < 90^\circ$, giving your answers to the nearest 0.1° . (6 marks)

No credit will be given for simply reading values from a graph.

(b) Describe a sequence of geometrical transformations that maps the curve $y = \sinx$ onto the curve $y = \sin(3x + 45^\circ)$. (3 marks)

Question Number and part	Solution	Marks	Total marks	Comments
6(a)	$3x + 45^\circ = 44.4(27)^\circ$ or $3x + 45^\circ = 180^\circ - "44.4(27)"^\circ$ or $3x + 45^\circ = -180^\circ - "44.4(27)"^\circ$ $x = -0.2^\circ$ $x = 30.2^\circ$ $x = -89.8^\circ$	B1 M1 M1 B1 A1ft A1ft	6	44.4(27) 180 – (cand's principal value) – 180 – (cand's principal value) Accept – 0.19 or better ft 'their' 44.4(27) Extra solutions withhold final A1
(b)	Translation – 45° parallel to x -axis Stretch sf $\frac{1}{3}$ parallel to x -axis Correct order Alternative Stretch sf $\frac{1}{3}$ parallel to x -axis followed by Translation – 15° parallel to x -axis	B1 B1 B1	3	Generous for first B1 earned Translation with correct details Stretch with correct details Correct order sc if 0/3 give max B1 for both stretch and translation with no/wrong details.
	Total		9	

(a) (i) Express $\sin^2 x$ in terms of $\cos x$. (1 mark)

(ii) By writing $\cos x = y$, show that the equation

$$7 \cos x + 2 - 4 \sin^2 x = 0$$

is equivalent to

$$4y^2 + 7y - 2 = 0 \quad (2 \text{ marks})$$

(b) Solve the equation $4y^2 + 7y - 2 = 0$. (2 marks)

(c) Hence, solve the equation

$$7 \cos x + 2 - 4 \sin^2 x = 0$$

giving all solutions to the nearest 0.1° in the interval $0^\circ < x < 360^\circ$.

No credit will be given for simply reading values from a graph. (3 marks)

5(a)(i)	$\sin^2 x = 1 - \cos^2 x$	B1	1	
(ii)	Attempt to substitute for $\cos x$ & their (i) $\Rightarrow 4y^2 + 7y - 2 = 0$	M1 A1	 2	$7y + 2 - 4(1 - y^2)$ ag Watch printed answer (must have = 0) Award SC1 for answer involving $\cos x$
(b)	Use of quadratic equation formula or attempt to factorise $\Rightarrow (y =) -2, \frac{1}{4}$	M1 A1	 2	$(4y - 1)(y + 2) = 0$ Award these marks if working with $\cos x$ and not y
(c)	$\cos^{-1}(0.25)$ 75.5° 284.5°	M1 A1 A1✓	 3	Attempt to inverse cosine their y value $360^\circ - \text{"their P.V"}$
Total			8	

