7 The equations

$$x + y - 2z = 2$$
$$3x - y + 6z = 2$$
$$6x + 5y - 9z = k$$

represent three planes, where k is a constant.

(a) Show that this system of equations does not have a unique solution. *(2 marks)*

(b) Prove that this system is consistent provided $k = 11$. *(4 marks)*

(c) (i) Find the solution to this system in the case when $k = 11$. *(4 marks)*

(ii) Interpret this solution with reference to the three planes. *(1 mark)*

7 (a)	$\begin{vmatrix} 1 & 1 & -2 \\ 3 & -1 & 6 \\ 6 & 5 & -9 \end{vmatrix} = 9 + 36 - 30 - 12 - 30 + 27$	M1		
	$= 0$	A1	2	N.B. Candidates may conclude this after work on augmented matrix (e.g.) or by $21R_1 + R_2 = 4R_3$, etc.
(b)	$\left[\begin{array}{ccc\|c} 1 & 1 & -2 & 2 \\ 3 & -1 & 6 & 2 \\ 6 & 5 & -9 & k \end{array}\right] \rightarrow$	M1		
	$\left[\begin{array}{ccc\|c} 1 & 1 & -2 & 2 \\ 0 & -4 & 12 & -4 \\ 0 & -1 & 3 & k-12 \end{array}\right]$	A1 A1		R_2 R_3
	Consistency provided $k - 12 = \frac{1}{4}(-4)$			
	$\Rightarrow k = 11$	A1	4	**ag** N.B. Give 3 + A0 for showing that $k = 11$ $\Rightarrow$ consistency rather than vice versa
(c)	(i) $y - 3z = 1$ from R_3 (e.g.)	B1		
	Let $z = \lambda$	M1		
	$\Rightarrow y = 3\lambda + 1$	A1		
	And $x = 2 + 2z - y = 1 - \lambda$	A1	4	
	(ii) Line of intersection	B1	1	N.B. Since candidates can get several results from the same piece of working (on the augmented matrix), assign marks for whichever section would credit them most favourably, irrespective of which section they believe themselves to be answering.
	Total		**11**	

5 Three simultaneous equations are

$$\begin{aligned} x - 3y + 2z &= 3 \\ x + y + az &= b \\ x - 2y + z &= 2, \end{aligned}$$

where a and b are constants.

(a) In the case where $a \neq -2$, solve the equations in terms of a and b. *(7 marks)*

(b) Give, with reasons, a geometrical interpretation of the planes represented by these three equations in the case where $a = -2$ and $b \neq -1$. *(3 marks)*

5 (a)	$4y+(a-2)z=b-3$ $y-z=-1$	M1A1		M1 for elimination of one variable
	$(a+2)z=b+1$	M1		
	$z=\dfrac{b+1}{a+2}$	A1		M1 for elimination of two variables
	$x=\dfrac{b+1}{a+2}$, $y=\dfrac{b-a-1}{a+2}$	M1M1A1	7	M2 for complete solution [single fraction for A1s]
(b)	No solution	B1		
	Planes not parallel	B1		
	So a prism	B1	3	allow diagram
	Total		**10**	

6 A matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 3 & 1 & 8 \\ 2 & -1 & 5 \\ 1 & 2 & a \end{bmatrix}.$$

(a) Find det **M** in terms of a. *(3 marks)*

(b) Find the value of a for which the matrix **M** is singular. *(1 mark)*

(c) (i) In the case $a = 2$, find $\mathbf{M}^{-1}$. *(6 marks)*

(ii) Hence, or otherwise, solve

$$3x + y + 8z = 3$$
$$2x - y + 5z = 0$$
$$x + 2y + 2z = 2.$$

(4 marks)

6 (a)	$3(-a-10)-(2a-5)+8(4+1)$	M1A1		M attempt; A correct unsimplified
	$15-5a$	A1	3	CAO
(b)	$a=3$	B1√	1	ft $\Delta = 0$
(c)(i)	$\begin{bmatrix} -12 & -1 & 5 \\ -14 & -2 & 5 \\ 13 & -1 & -5 \end{bmatrix}$	M1 A1		Finding 2×2 determinants (co-factors) Any one correct row/column
	Det $=5$	B1		ft (a) wrong or correct having started again
	$\frac{1}{5}\begin{bmatrix} -12 & 14 & 13 \\ 1 & -2 & 1 \\ 5 & -5 & -5 \end{bmatrix}$	M1 M1 A1	6	signs Transpose CAO
(ii)	$\frac{1}{5}\begin{bmatrix} -12 & 14 & 13 \\ 1 & -2 & 1 \\ 5 & -5 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$	M2		
	$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$	A1√A1	4	A1 any one correct (ft) A1 all CAO
	Alternative 1 to (c)(ii) 3 eqns $\to 2 \to 1 \to$ Answers	*(M1)* *(M1)* *(A1)* *(A1)*		3 eqns $\to 2$ 2 eqns $\to 1$ any one correct all CAO
	Alternative 2 to (c)(ii) Cramer's Rule $x = \frac{\Delta x}{\Delta}$ etc	*(M1)*		
	$x=-2,\ y=1,\ z=1$	*(A1A1A1)*		
	Alternative 3 to (c)(ii) Gaussian Elimination	*(M1A1A1)* *(A1)*		
	Total		**14**	

3 Two planes are represented by the equations

$$x + y + z = 3,$$
$$5x + y + 3z = 29.$$

(a) Find the equations of the line of intersection of the planes, giving your answer in the form

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}.$$

(6 marks)

(b) Show that this line also lies in the plane with equation

$$4x - 2y + z = 33.$$

(3 marks)

3 (a)	Elimination of 1 variable	M1		
	Elimination of another	M1A1		e.g. $\begin{cases} x-y=10 \\ 2x+z=13 \end{cases}$
	$x = y + 10 = \dfrac{z-13}{-2}$	M1A1A1	6	M1 for relationship in required form
(b)	e.g. Substitute $x=\lambda$, $y=\lambda-10$, $z=13-2\lambda$	M1		
	$4\lambda - 2(\lambda-10) + 13 - 2\lambda = 33$	M1A1	3	AG
	Total		**9**	