D2 Simplex algorithm Challenge

Challenge 1

A linear programming problem in x and y is to be solved. Part of the initial tableau is given below.

x	y	r	s	t	
4	3	1	0	0	33
–1	1	0	1	0	4
2	5	0	0	1	27

(a) In addition to $x \geqslant 0$ and $y \geqslant 0$, write down the **three** inequalities in this problem.
(2 marks)

(b) (i) The objective function $P = 2x + 2y$ is to be maximised.
Solve this linear programming problem using the simplex algorithm, by initially using a value in the x column as the pivot. (You do **not** require more than two iterations.)
(7 marks)

(ii) State your final values of P, x and y. *(2 marks)*

Challenge 2

(a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x + 5y + 3z$

subject to $8x + 5y + 2z \leqslant 3$

$4x + 6y + 9z \leqslant 2$

and $x \geqslant 0, y \geqslant 0, z \geqslant 0$ *(2 marks)*

(b) Solve the problem using the Simplex algorithm, giving your answers as exact fractions.
(9 marks)

Final Challenge

The simplex method has been applied to a linear programming problem concerning an objective function P in two variables, x and y. The initial tableau T_0 and the tableau T_1, after one iteration of the simplex method, are given by:

T_0

P	x	y	s	t	u	
1	−1	−2	0	0	0	0
0	−1	1	1	0	0	5
0	1	1	0	1	0	15
0	1	−3	0	0	1	3

T_1

P	x	y	s	t	u	
1	0	−5	0	0	1	3
0	0	−2	1	0	1	8
0	0	4	0	1	−1	12
0	1	−3	0	0	1	3

(a) (i) Apply one further iteration of the simplex method to give a new tableau T_2. *(5 marks)*

(ii) Explain how you know that the maximum value of P has not yet been reached. *(1 mark)*

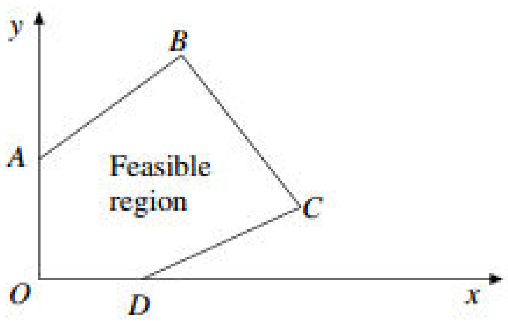
(b) A further iteration of the simplex method leads to tableau T_3.

T_3

P	x	y	s	t	u	
1	0	0	$\frac{1}{2}$	$1\frac{1}{2}$	0	25
0	0	0	2	1	1	28
0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	10
0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	5

(i) State the maximum value of P and the values of x and y for which this maximum is reached. *(2 marks)*

(ii) The figure shows a sketch of the feasible region of the linear programming problem.



For each of the tableaux T_0, T_1, T_2 and T_3 state which of the points O, A, B, C or D it represents. *(3 marks)*

(iii) Explain how the original linear programming problem could have been solved by the simplex method with fewer than 3 iterations. *(2 marks)*