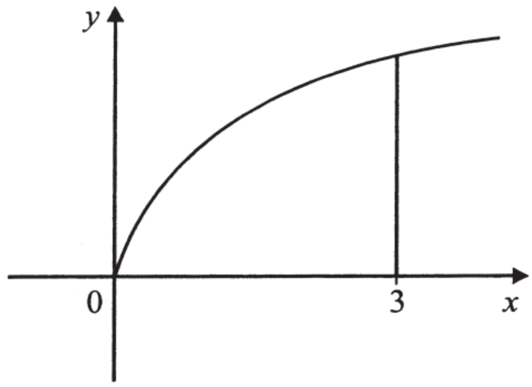
FP2 - Revolutions

Challenge 1



The diagram shows a sketch of the curve $y = 2\sqrt{x}$.

The arc of the curve between $x = 0$ and $x = 3$ is rotated through 2π radians about the x – axis.

(a) Show that S, the surface area generated, is given by

$$S = 4\pi \int_0^3 \sqrt{1 + x}\,dx.$$ *(5 marks)*

(b) Hence evaluate S. *(3 marks)*

Challenge 2

A curve has equation

$$y = \sinh^2 x.$$

(a) Show that

$$1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2 2x. \qquad \textit{(2 marks)}$$

The arc of the curve between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis.

(b) (i) Show that S, the area of the curved surface generated, is given by

$$S = \pi \int_0^1 (\cosh 2x - 1) \cosh 2x \, dx. \qquad \textit{(3 marks)}$$

(ii) Hence find S, giving an exact answer in terms of hyperbolic functions. *(4 marks)*

Challenge 3

A curve C has equation

$$y = \ln(1 - x^2), \qquad 0 \leqslant x < 1.$$

(a) Show that

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(\frac{1 + x^2}{1 - x^2}\right)^2.$$

(6 marks)

(b) Use the result

$$\frac{1 + x^2}{1 - x^2} = \frac{2}{1 - x^2} - 1$$

to show that the length of the arc of C between the points where $x = 0$ and $x = p$ is

$$2\tanh^{-1} p - p.$$

(4 marks)

Final Challenge

(a) Evaluate:

(i) $\int \cosh^2 x \, dx$; *(3 marks)*

(ii) $\int x \cosh x \, dx$. *(3 marks)*

(b) A curve C is given parametrically by the equations

$$x = \cosh t + t, \qquad y = \cosh t - t.$$

Express

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

in terms of $\cosh t$. *(5 marks)*

(c) (i) The arc of C from $t = 0$ to $t = 1$ is rotated through 2π radians about the x-axis.

Show that S, the area of the curved surface generated, is given by

$$S = 2\pi\sqrt{2}\int_0^1 (\cosh t - t)\cosh t \, dt.$$ *(1 mark)*

(ii) Hence find S, leaving your answer in terms of hyperbolic functions. *(4 marks)*