

Core 1 – Quadratic Equations

Challenge 1

- (a) (i) Express $x^2 + 12x + 11$ in the form $(x + a)^2 + b$, finding the values of a and b . (2 marks)
- (ii) State the minimum value of the expression $x^2 + 12x + 11$. (1 mark)
- (b) Determine the values of k for which the quadratic equation

$$x^2 + 3(k - 2)x + (k + 5) = 0$$

has equal roots.

(4 marks)



Challenge 2

The quadratic equation

$$x^2 + (3 - k)x + 5 - k^2 = 0$$

is to be considered for different values of the constant k .

(a) When $k = 7$:

(i) show that $x^2 - 4x - 44 = 0$; *(1 mark)*

(ii) find the roots of this equation, giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. *(2 marks)*

(b) When the quadratic equation $x^2 + (3 - k)x + 5 - k^2 = 0$ has equal roots:

(i) show that $5k^2 - 6k - 11 = 0$; *(3 marks)*

(ii) hence find the possible values of k . *(2 marks)*



Challenge 3

- (a) (i) Express $x^2 + 8x + 11$ in the form $(x + p)^2 + q$. (2 marks)
- (ii) Hence, or otherwise, find the coordinates of the minimum point of the curve with equation $y = x^2 + 8x + 11$. (2 marks)
- (b) Describe in detail the geometrical transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 11$. (3 marks)
- (c) Determine the condition on k for which the equation

$$x^2 + 8x + 11 - k = 0$$

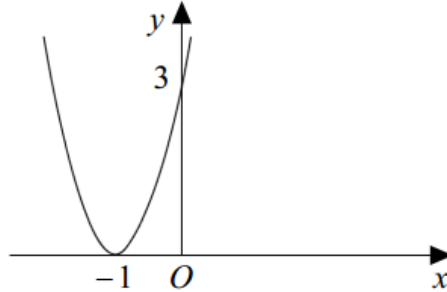
has no real solutions.

(3 marks)



Final Challenge

The graph of $y = 3(x + 1)^2$ is sketched below.



- (a) Describe fully a sequence of geometrical transformations that would map the graph of $y = x^2$ onto the graph of $y = 3(x + 1)^2$. (4 marks)

- (b) (i) Express $3(x + 1)^2$ in the form $px^2 + qx + r$. (1 mark)

- (ii) Find the gradient of the curve with equation $y = 3(x + 1)^2$ at the point where $x = 4$. (3 marks)

- (c) (i) Show that the curve with equation $y = 3(x + 1)^2$ and the line with equation $y = kx - 9$ intersect when

$$3x^2 + (6 - k)x + 12 = 0 \quad (1 \text{ mark})$$

- (ii) Find the values of k for which the quadratic equation

$$3x^2 + (6 - k)x + 12 = 0$$

has equal roots. (4 marks)

- (iii) State the geometrical relationship between the line $y = kx - 9$ and the curve $y = 3(x + 1)^2$ for these values of k . (1 mark)