

A group of five pupils have to colour in some pictures. There are five coloured crayons available, Orange (O), Red (R), Yellow (Y), Green (G) and Purple (P). The five pupils have each told their teacher their first and second choice of coloured crayons.

Pupil	First choice	Second choice
Alison (A)	Orange	Yellow
Brian (B)	Orange	Red
Carly (C)	Yellow	Purple
Danny (D)	Red	Purple
Emma (E)	Purple	Green

- (a) Show this information on a bipartite graph. (2 marks)
- (b) Initially the teacher gives pupils A , C , D and E their first choice of crayons. Demonstrate, by using an algorithm from this initial matching, how the teacher can give each pupil either their first or second choice of coloured crayons. (4 marks)

Q	Solution	Marks	Total	Comments
2(a)		M1 A1	2	bipartite graph
(b)	<p>Initial $A \rightarrow O$ $C \rightarrow Y$ $D \rightarrow R$ $E \rightarrow P$</p> <p>Alternating path $B-R = D-P = E-G$ or $B \rightarrow O, A \rightarrow Y, C \rightarrow P$ (break through) $E \rightarrow G, (D \rightarrow R)$</p> <p>Complete match: $A \rightarrow O$ or $A \rightarrow Y$ $B \rightarrow R$ $B \rightarrow O$ $C \rightarrow Y$ $C \rightarrow P$ $D \rightarrow P$ $D \rightarrow R$ $E \rightarrow G$ $E \rightarrow G$</p>	M1 A1 B1	4	must start from B or G
Total			6	

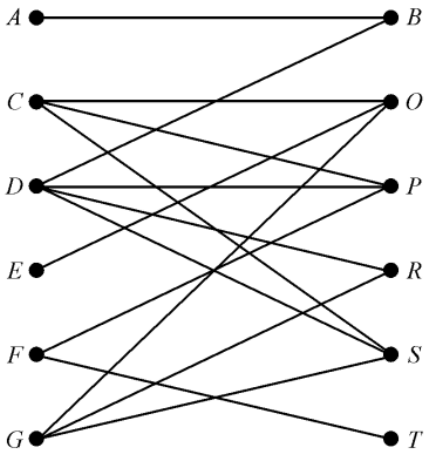
Six children are going to eat some fruit pastilles. There are six pastilles: Blackcurrant (B), Orange (O), Plum (P), Raspberry (R), Strawberry (S) and Tango (T). The six children will eat only certain flavours.

Name	Flavours
Alison (A)	B
Chris (C)	O, P, S
Derek (D)	B, P, R, S
Eddie (E)	O
Freda (F)	P, T
Gemma (G)	O, R, S

(a) Show this information on a bipartite graph. (2 marks)

(b) Initially, Chris chooses Orange, Derek chooses Blackcurrant, Freda chooses Plum and Gemma chooses a Strawberry pastille.

Demonstrate, by using an alternating path from this initial matching, how each child can get a pastille that they will eat. (5 marks)

<p>2 (a)</p>		<p>M1 A1</p>	<p>2</p>	<p>Bipartite All correct</p>
<p>(b)</p>	<p>$A \rightarrow B, D \rightarrow (R)$</p> <p>$E \rightarrow O, C \rightarrow P, F \rightarrow T$</p> <p>Complete match</p> <p>$A \rightarrow B$ $C \rightarrow P$ $D \rightarrow R (S)$ $E \rightarrow O$ $F \rightarrow T$ $G \rightarrow S (R)$ or $C \rightarrow S, D \rightarrow P, G \rightarrow R$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>B1</p>	<p>5</p>	<p>OE Starting (A, E) or (R, T)</p> <p>2nd application complete path</p>
Total			7	

Six people A, B, C, D, E and F are to be matched to six tasks 1, 2, 3, 4, 5 and 6. The following table shows the tasks that each of the people is able to undertake.

Person	Task(s)
A	1, 2, 3
B	1, 2, 5
C	2, 3, 4
D	3, 4, 6
E	3
F	2

(a) Show this information on a bipartite graph. *(2 marks)*

(b) The following is the initial matching:

$$A - 2, \quad B - 1, \quad C - 3, \quad D - 4.$$

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated a task that they can undertake. *(5 marks)*

Q	Solution	Marks	Total	Comments
1 (a)		M1 A1	2	Bipartite graph
(b)	<p>(Initial $A \rightarrow 2, B \rightarrow 1, C \rightarrow 3, D \rightarrow 4$)</p> <p>$\left\{ \begin{array}{l} \therefore E \rightarrow 3 \rightarrow C \rightarrow 2 \rightarrow A \rightarrow 1 \rightarrow B \rightarrow 5 \\ \text{then } F \rightarrow 2 \rightarrow C \rightarrow 4 \end{array} \right.$</p>	M1A1 M1A1		<p>1st path</p> <p>2nd path</p> <p>or $\left\{ \begin{array}{l} E \rightarrow 3 \rightarrow C \rightarrow 4 \rightarrow D \rightarrow 6 \quad \text{M1A1} \\ \text{then } F \rightarrow 2 \rightarrow A \rightarrow 1 \rightarrow B \rightarrow 5 \quad \text{M1A1} \end{array} \right.$</p> <p>or $\left\{ \begin{array}{l} F \rightarrow 2 \rightarrow A \rightarrow 3 \rightarrow C \rightarrow 4 \rightarrow D \rightarrow 6 \quad \text{M1A1} \\ \text{then } E \rightarrow 3 \rightarrow A \rightarrow 1 \rightarrow B \rightarrow 5 \quad \text{M1A1} \end{array} \right.$</p>
	Match: A1, B5, C4, D6, E3, F2	B1	5	
		Total	7	

Four people A, B, C and D are to be matched to four tasks 1, 2, 3 and 4.

A bipartite graph showing the possible allocation of people to jobs is shown in **Figure 1**.

An initial matching is shown in **Figure 2**.

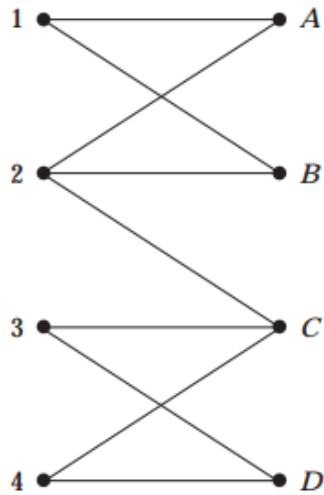


Figure 1

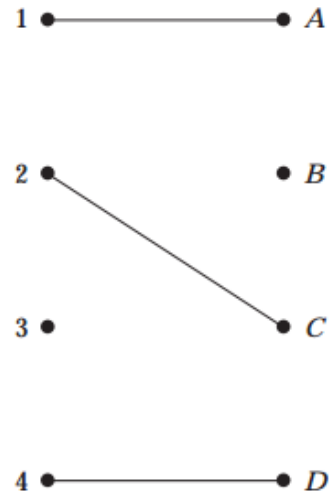


Figure 2

- (a) Write down an adjacency matrix that represents the bipartite graph shown in **Figure 1**. (2 marks)
- (b) There are four distinct alternating paths that can be generated from the initial matching shown in **Figure 2**.

One of the paths is

$$3 - C - 2 - A - 1 - B,$$

which produces the following complete matching

$$1 - B, \quad 2 - A, \quad 3 - C, \quad 4 - D.$$

- (i) Use the maximum matching algorithm from the initial matching to find another maximum matching, listing the complete matching generated. (3 marks)
- (ii) Find the remaining two alternating paths and list the complete matchings generated in each case. (4 marks)

Q	Solution	Marks	Total	Comments
1 (a)	$ \begin{array}{cccc} & A & B & C & D \\ 1 & \left(\begin{array}{cccc} 1 & 1 & 0 & 0 \end{array} \right) \\ 2 & \left(\begin{array}{cccc} 1 & 1 & 1 & 0 \end{array} \right) \\ 3 & \left(\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array} \right) \\ 4 & \left(\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array} \right) \end{array} $	M1A1	2	
(b)(i)	$3 - C - 2 - B$ Match (1A 2B 3C 4D)	M1A1 B1	3	or answers from (ii)
(ii)	$3 - D - 4 - C - 2 - B$ Match (1A 2B 3D 4C)	M1 B1		or in diagram
	$3 - D - 4 - C - 2 - A - 1 - B$ Match (1B 2A 3D 4C)	M1 B1	4	or in diagram
	Total		9	