The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(Total 5 marks)

$\frac{dy}{dx} = \frac{1}{x}$	M1 A1	
At $x = 3$, gradient of normal $= \frac{-1}{\frac{1}{3}} = -3$	M1	
$y - \ln 1 = -3(x - 3)$	M1	
$y = -3x + 9$	A1	5

[5]

(a) Sketch on one pair of axes the graphs of

$$y = 6 - x \text{ and } y = \ln x .$$ *(1 mark)*

(b) Hence state the number of roots of the equation

$$6 - x = \ln x .$$ *(1 mark)*

(c) By considering values of the function f, where

$$f(x) = 6 - x - \ln x ,$$

(i) show that the equation in part (b) has a root α such that

$$4 < \alpha < 5 ,$$ *(2 marks)*

(ii) determine whether α is closer to 4 or to 5. *(2 marks)*

2 (a)	Sketch	B1	1	condone no scales for x and/or y
(b)	One root	B1	1	
(c)(i)	$f(4) \approx -0.61$, $f(5) \approx -0.61$	B1		numerical evidence needed; allow clear comparison of values of $6 - x$ and $\ln x$
	Change of sign $\Rightarrow 4 < \alpha < 5$	E1	2	AG; change of sign may not be mentioned but conclusion must be drawn
(ii)	$f(4.5) \approx -0.004$	B1		evidence needed; allow comparison
	Negative value $\Rightarrow \alpha$ nearer to 4	E1F	2	f.t positive value for f(4.5)
	Total		**6**	

(a) (i) Draw on the same diagram sketches of the graphs with equations

$$y = 5e^{2x} \quad \text{and} \quad y = \frac{4}{x} \quad \text{for} \quad x > 0.$$

(2 marks)

(ii) Explain why this diagram shows that, for $x > 0$, the equation

$$5e^{2x} - \frac{4}{x} = 0$$

has just one root, α, and show that $0.3 < \alpha < 0.4$. *(2 marks)*

(b) Show, using calculus, that $y = 5e^{2x} - \frac{4}{x}$ is an increasing function of x for $x > 0$.

(3 marks)

(c) Show that the area of the region enclosed by the curve $y = 5e^{2x} - \frac{4}{x}$, the x-axis, and the lines $x = \frac{1}{2}$ and $x = 2$ can be expressed in the form

$$\frac{5}{2}\left(e^4 - e\right) - k \ln 2$$

for some positive integer k whose value is to be determined. *(5 marks)*

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7 (a) (i)	**Sketches of $y = 5e^{2x}$ and $y = \frac{4}{x}$ for $x \geq (>) 0$**	**B1 B1**	**Ignore $x < 0$**	**2**
(ii)	**Only one point of intersection noted or stated**	**E1**		
	$f(0.3) \approx -4.22 < 0$ and $f(0.4) \approx 1.13 > 0$	**B1**	**No explanation required**	**2**
(b)	**$y' = 10e^{2x} + \frac{4}{x^2}$**	**B1 B1**	**One for each term correct**	
	$y' > 0$ stated or explained (for all $x > 0$)	**B1**	**FT $Ae^{kx} + \frac{B}{x^2}$ $(A, B > 0)$**	**3**
(c)	**$\int\left(5e^{2x} - \frac{4}{x}\right) dx$ attempted**	**M1**		
	$= \frac{5}{2}e^{2x} - 4\ln x$	**A1 A1**		
	$= 2.5(e^4 - e) - 4(\ln 2 - \ln ½)$	**M1**	**Both limits substd. in FT expression with 2 variable terms**	
	$= 2.5(e^4 - e) - 8\ln 2$ i.e $k = 8$	**A1**	**CAO**	**5**

(a) (i) Draw on the same diagram sketches of the graphs with equations

$$y = x - 2 \text{ and } y = 2 \ln x \text{ for } x > 0$$

(2 marks)

(ii) Hence state the number of roots of the equation

$$x - 2 = 2 \ln x, \qquad x > 0$$

(1 mark)

(b) The curve, C, with equation

$$y = x - 2 - 2 \ln x, \qquad x > 0$$

has only one stationary point.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Show that the y-coordinate of the stationary point is $-\ln 4$. *(3 marks)*

(iii) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(iv) Hence show that the stationary point is a minimum. *(1 mark)*

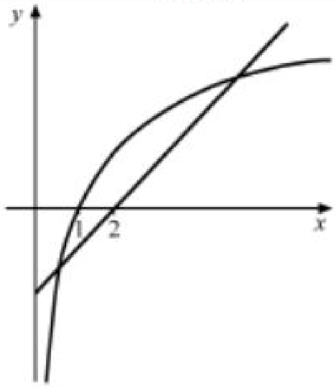
(c) The vertical lines $x=6$ and $x=7$ meet the curve C at points P and Q respectively.

(i) Show that the y-coordinate of P is $4 - \ln 36$. *(2 marks)*

(ii) The area of the trapezium bounded by the lines PQ, $x=6$, $x=7$ and the x-axis is A square units. Show that

$$A = \frac{p}{2} - \ln q$$

stating the values of the positive integers p and q. *(3 marks)*

Question	Solution	Marks	Total	Comments
7 a)(i)		B2,1	2	B2 graphs correct with details (B1 for correct shape of $y = 2\ln x$)
(ii)	2 roots	B1	1	cao (dependent on at least B1 in (i) and no obvious wrong reasoning)
(b)(i)	$y'(x) = 1 - \frac{2}{x}$	M1 A1	2	Clear attempt to differentiate
(ii)	At stationary point $y'(x) = 0 \Rightarrow 1 - \frac{2}{x} = 0$ $\Rightarrow x = 2$	M1		Attempts to solve $y'(x) = 0$
	When $x = 2,\ y = -2\ln 2$	A1		
	$y = -\ln 2^2 = -\ln 4$	A1	3	ag
(iii)	$\frac{d^2y}{dx^2} = \frac{2}{x^2}$	M1 A1	2	Clear attempt to differentiate $y'(x)$
(iv)	When $x = 2$, $y''(x) > 0 \Rightarrow$ stationary point is minimum	B1ft	1	ag
(c)(i)	$y(6) = 4 - 2\ln 6$	M1		Attempts to find y when $x = 6$
	$= 4 - \ln 6^2 = 4 - \ln 36$	A1	2	ag
(ii)	$A = \frac{1}{2}[y(6) + y(7)] \times 1$	M1		
	$= \frac{1}{2}[4 - \ln 36 + 5 - \ln 49]$			
	$= \frac{1}{2}[9 - \ln(36 \times 49)]$ or			
	$= \frac{1}{2}[9] - \ln 6 - \ln 7$	m1		Valid use of a law of logs
	$= \frac{9}{2} - \ln 42$ ie $p = 9;\ q = 42$	A1	3	
	Total		**16**	