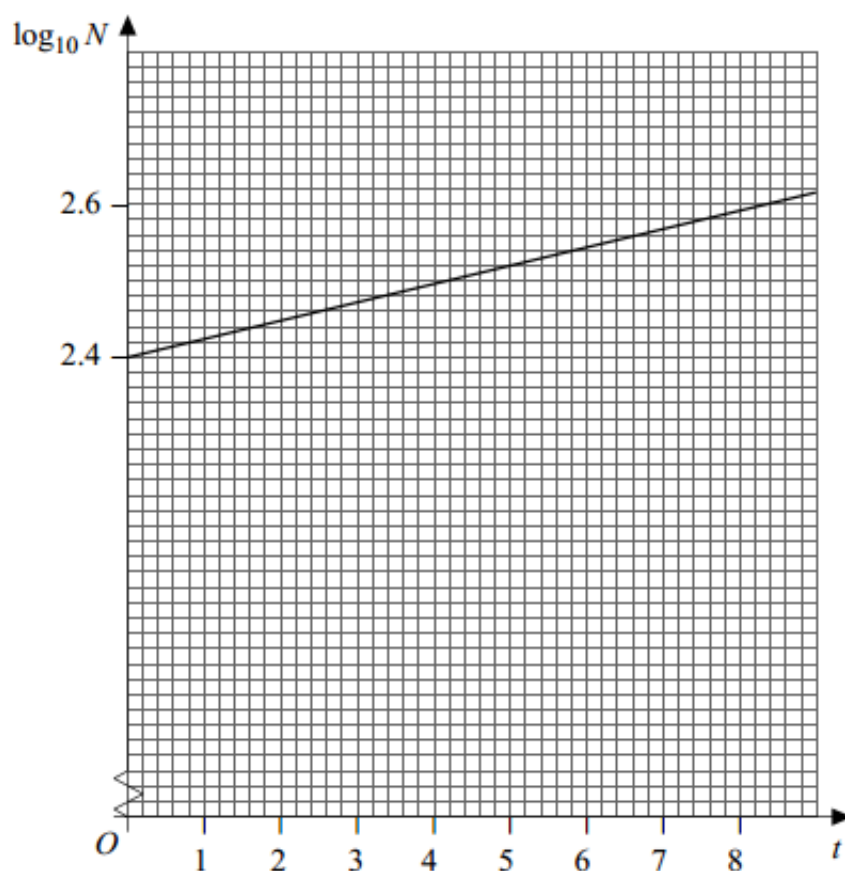


FP1 – Linear Laws Challenge

Challenge 1

A mathematical model is required to estimate the number, N , of a certain strain of bacteria in a test tube at time t hours after a certain instant.

After values of $\log_{10} N$ are plotted against t , a straight line graph can be drawn through the points as shown below.



- (a) Use the graph to estimate the number of bacteria when $t = 5$. (3 marks)
- (b) The graph would suggest that N and t are related by an equation of the form

$$N = a \times b^t$$

where a and b are constants.

- (i) Express $\log_{10} N$ in terms of $\log_{10} a$, $\log_{10} b$ and t . (2 marks)
- (ii) Use the graph to determine the values of a and b , giving your answers to 3 significant figures. (4 marks)
- (c) Suggest why the model $N = a \times b^t$ is likely to give an overestimate of the number of bacteria in the test tube for large values of t . (1 mark)

Challenge 2

[A sheet of graph paper is provided for use in this question.]

The variables T and L satisfy a relationship of the form $T = aL^b$, where a and b are constants.

Measurements of T for given values of L gave the following results.

L	2	3	4	5	6
T	5.62	6.94	8.03	8.98	9.97

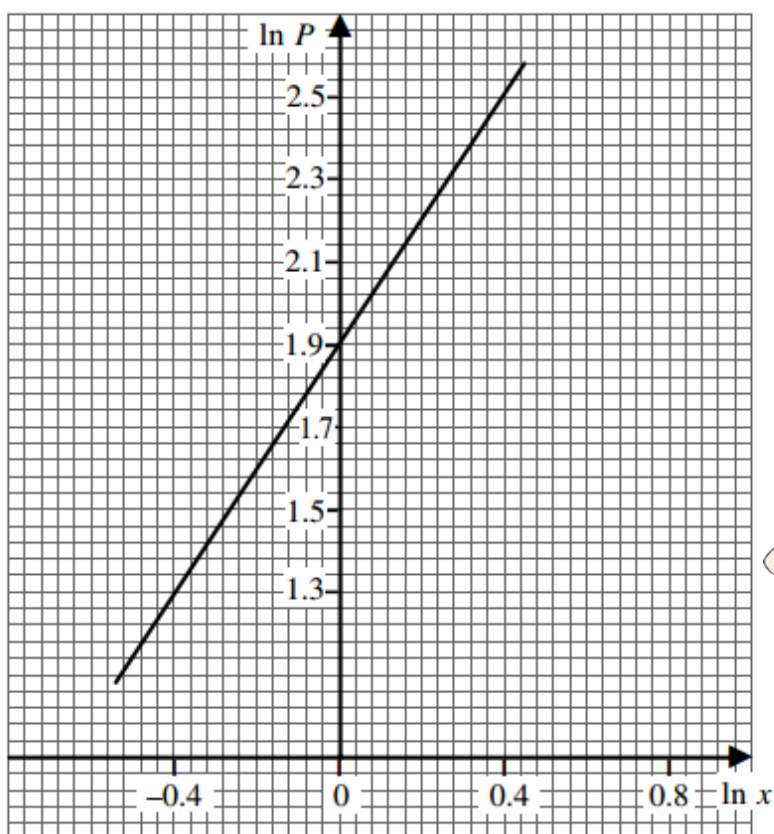
- (a) Express $\ln T$ in terms of a , b and $\ln L$. (1 mark)
- (b) Plot $\ln T$ against $\ln L$ on graph paper. (3 marks)
- (c) Draw a suitable straight line to illustrate the relationship between the data. (1 mark)
- (d) Use your line to estimate
- (i) the value of L when $T = 8.50$ giving your answer to two significant figures, (2 marks)
- (ii) the values of a and b , giving your answers to two significant figures. (4 marks)



Challenge 3

A mathematical model is used by an astronomer to investigate features of the moons of a particular planet. The mean distance of a moon from the planet, measured in millions of kilometres, is denoted by x , and the corresponding period of its orbit is P days.

The model assumes that the graph of $\ln P$ against $\ln x$ is the straight line drawn below.



(a) Use the graph to estimate the period of the orbit of a moon for which $x = 1.43$. (3 marks)

(b) The graph would suggest that P and x are related by an equation of the form

$$P = kx^\alpha$$

where k and α are constants.

(i) Express $\ln P$ in terms of $\ln k$, $\ln x$ and α . (1 mark)

(ii) Use the graph to determine the values of k and α , giving your answers to 2 significant figures. (4 marks)

Final Challenge

[A sheet of graph paper is supplied for use in this question.]

The energy, E , lost in a cycle of magnetization of a transformer core is thought to relate to the flux density, B , by a law of the form $E = kB^\alpha$ where k and α are constants.

- (a) Express $\ln E$ in terms of $\ln k$, α and $\ln B$. (1 mark)

For a given material, the values of B and E in appropriate units are:

B	3.16	9.56	18.3	29.0	41.4
E	1	2	3	4	5

- (b) Plot $\ln E$ against $\ln B$ on graph paper. (3 marks)
- (c) Draw a suitable straight line to illustrate the relationship between the data. (1 mark)
- (d) Use your line to estimate:
- (i) the value of E when $B = 25.5$ giving your answer to 2 significant figures; (3 marks)
 - (ii) the values of k and α , giving your answers to 2 significant figures. (4 marks)

