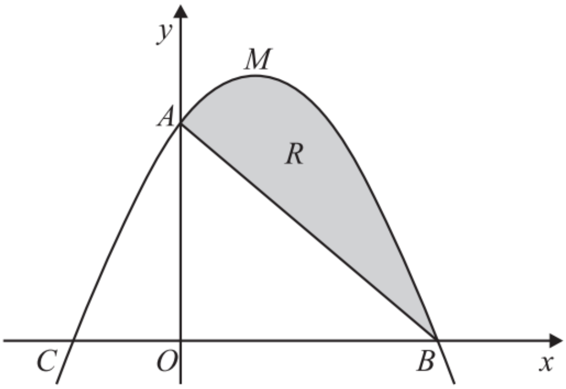
Core 1 - Integration

Challenge 1



The curve with equation $y = 12 + 4x - x^2$ cuts the y-axis at A, the positive x-axis at B and the negative x-axis at C as shown in the diagram. The point O is the origin and the maximum point of the curve is M. The shaded region R is bounded by the line AB and the curve.

The point B has coordinates (6, 0).

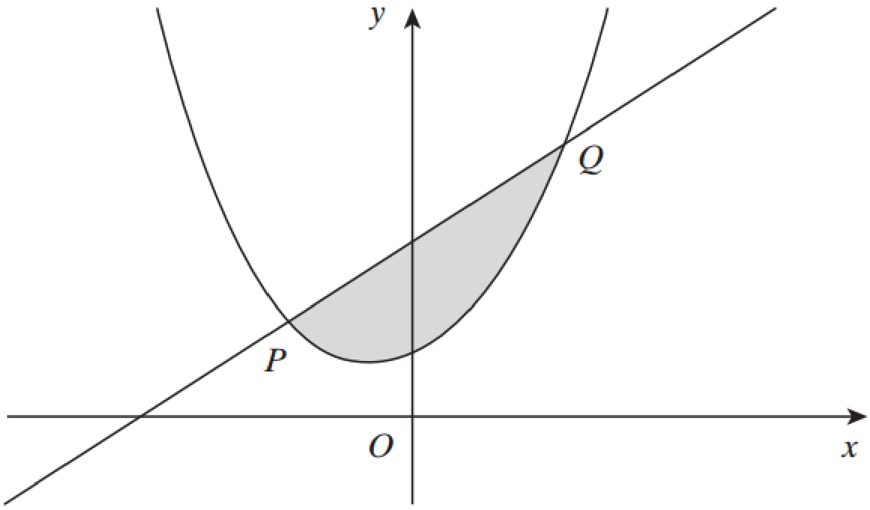
(a) Show that $x = 2$ at the point M. *(3 marks)*

(b) Find the coordinates of C. *(2 marks)*

(c) Show that triangle OAB and the region R have equal areas. *(6 marks)*

Challenge 2

The line $y = 2x + 5$ intersects the curve $y = x^2 + 2x + 2$ at the points P and Q, as shown in the diagram.



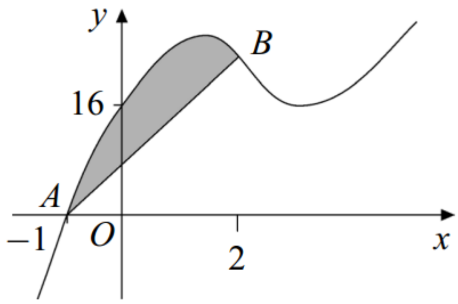
(a) Find the coordinates of P and Q, giving your answers in surd form. *(4 marks)*

(b) Find the area of the shaded region, giving your answer in surd form. *(9 marks)*

Challenge 3

The curve with equation $y = x^3 - 6x^2 + 9x + 16$ is sketched below.

The curve crosses the x-axis at the point $A\,(-1, 0)$.



(a) (i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Hence find the x-coordinates of the stationary points of the curve. *(3 marks)*

(b) (i) Find $\displaystyle\int_{-1}^{2} (x^3 - 6x^2 + 9x + 16)\,\mathrm{d}x$. *(5 marks)*

(ii) The point $B\,(2, 18)$ lies on the curve. Find the area of the shaded region bounded by the curve and the line AB. *(3 marks)*

Final Challenge

The function f is defined for all values of x by

$$f(x) = x^3 - 7x^2 + 14x - 8.$$

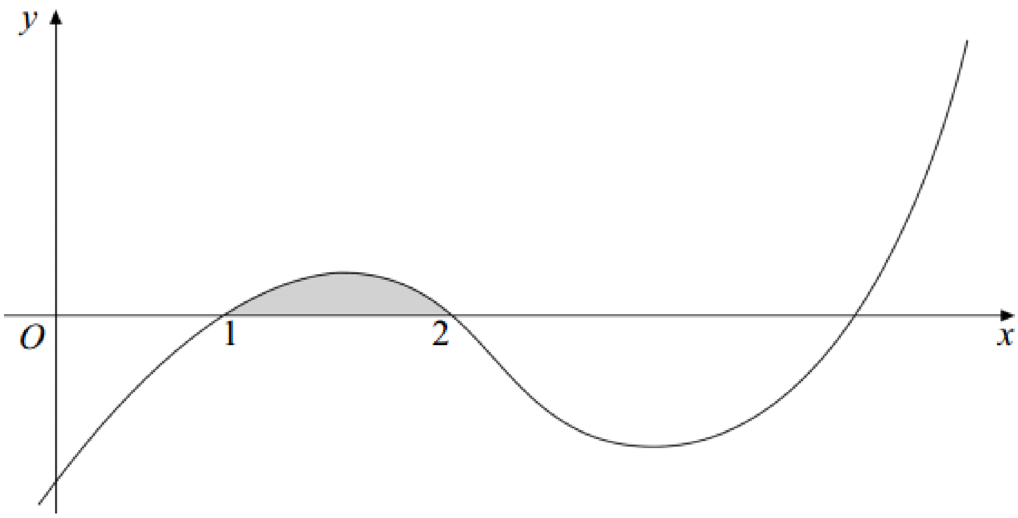
It is given that $f(1) = 0$ and $f(2) = 0$.

(a) Find the values of $f(3)$ and $f(4)$. *(2 marks)*

(b) Write $f(x)$ as a product of **three** linear factors. *(2 marks)*

(c) The diagram shows the graph of

$$y = x^3 - 7x^2 + 14x - 8.$$



(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) State, giving a reason, whether the function f is increasing or decreasing at the point where $x = 3$. *(2 marks)*

(iii) Find $\int (x^3 - 7x^2 + 14x - 8)\,dx$. *(3 marks)*

(iv) Hence find the area of the shaded region enclosed by the graph of $y = f(x)$, for $1 \leqslant x \leqslant 2$, and the x-axis. *(3 marks)*