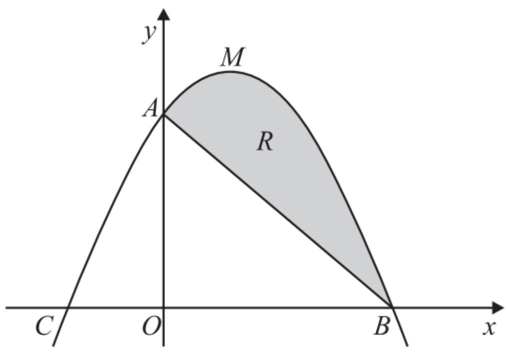
Integration



The curve with equation $y = 12 + 4x - x^2$ cuts the y-axis at A, the positive x-axis at B and the negative x-axis at C as shown in the diagram. The point O is the origin and the maximum point of the curve is M. The shaded region R is bounded by the line AB and the curve.

The point B has coordinates (6, 0).

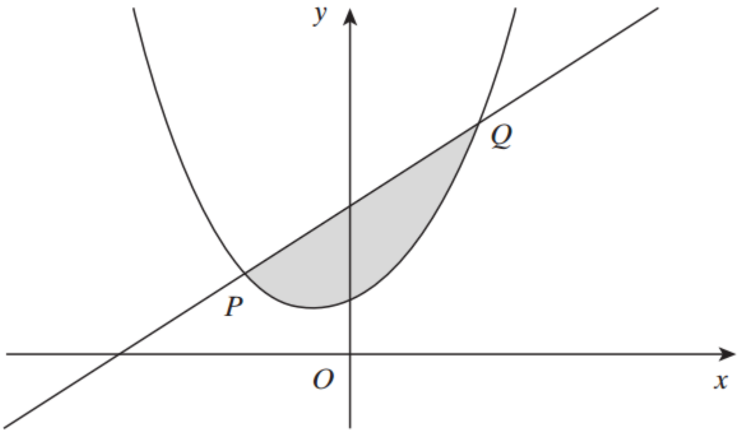
(a) Show that $x = 2$ at the point M. *(3 marks)*

(b) Find the coordinates of C. *(2 marks)*

(c) Show that triangle OAB and the region R have equal areas. *(6 marks)*

Question	Solution	Marks	Total	Comments
5 (a)	$\frac{dy}{dx} = 4 - 2x$	M1		At least one of the two correct
	At M, $\frac{dy}{dx} = 0$	m1		
	$4 - 2x = 0 \Rightarrow x = 2$	A1 cso	**(3)**	AG Completed convincingly
(b)	At C, $0=12+4x-x^2 \Rightarrow 0=(6-x)(2+x)$	M1		Factorisation or formula or use of sym of quadratic
	$x < 0$ at $C \Rightarrow x = \underline{-2;}$ $(C(-2,0))$	A1	**(2)**	
(c)	$A(0,12)$; area $\Delta OAB = 36$	B1		
	(Area $OAMB$) = $\int_0^6 12 + 4x - x^2 dx$	B1		Condone $\int_6^0$
	$= [12x + 2x^2 - \frac{1}{3}x^3]$	M1 A1		Integration; at least two correct All three correct
	…= 72+72−72 = 72	A1		
	Area of R=72−ΔOAB=36= area ΔOAB	A1 cso	**(6)**	AG Obtained convincingly
		TOTAL	**(11)**	

The line $y = 2x + 5$ intersects the curve $y = x^2 + 2x + 2$ at the points P and Q, as shown in the diagram.



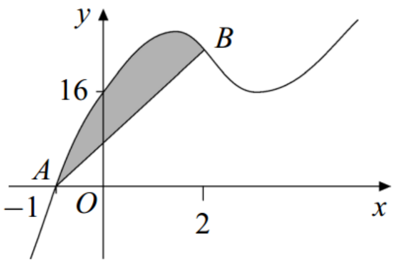
(a) Find the coordinates of P and Q, giving your answers in surd form. *(4 marks)*

(b) Find the area of the shaded region, giving your answer in surd form. *(9 marks)*

Q	Solution	Marks	Total	Comments
8 (a)	$x^2 + 2x + 2 = 2x + 5$	M1		
	$x = \pm\sqrt{3},\ \pm\dfrac{\sqrt{12}}{2},\ \ \pm 1.73$	M1A1		+1.73 M1M1A0A0
	$(\sqrt{3}, 2\sqrt{3}+5),\ \ (-\sqrt{3}, -2\sqrt{3}+5)$	A1	4	
(b)	Trapezium			
	$2\sqrt{3} \times 5$	B1M1		B1 for average height = 5
	$10\sqrt{3}$ or 17.3	A1		
	Under curve			
	$\int_{-\sqrt{3}}^{\sqrt{3}} x^2 + 2x + 2 \, dx$	B1√		√ on (a), allow 0, $\sqrt{3}$
	$\left[\frac{1}{3}x^3 + x^2 + 2x\right]$	M1A1		M1 for 2 correct
	$F(\sqrt{3}) - F(-\sqrt{3}) = 6\sqrt{3}$ or 10.4	M1A1		M1 for use of their limits following integration attempt
	Area $= 4\sqrt{3}$, single term	A1	9	
	Total		**13**	

The curve with equation $y = x^3 - 6x^2 + 9x + 16$ is sketched below.

The curve crosses the x-axis at the point $A\,(-1, 0)$.



(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence find the x-coordinates of the stationary points of the curve. *(3 marks)*

(b) (i) Find $\int_{-1}^{2} (x^3 - 6x^2 + 9x + 16)\,dx$. *(5 marks)*

(ii) The point $B\,(2, 18)$ lies on the curve. Find the area of the shaded region bounded by the curve and the line AB. *(3 marks)*

6(a)(i)	$\dfrac{dy}{dx} = 3x^2 - 12x + 9$	M1 A1 A1	 3	Attempt to differentiate; a power decreased by 1 Two terms correct All correct (withhold if $+c$ in answer)
(ii)	Putting candidate's $\dfrac{dy}{dx} = 0$ $3(x-1)(x-3)$ $x = 1, 3$	M1 m1 A1	 3	$3x^2 - 12x + 9 = 0$ Attempt to solve or factorise Both values and no others oe
(b)(i)	$\dfrac{x^4}{4} - 2x^3 + \dfrac{9}{2}x^2 + 16x$ $[4 - 16 + 18 + 32] - [\frac{1}{4} + 2 + 4\frac{1}{2} - 16]$ $= 47.25$ oe	M1 A1 A1 m1 A1	 5	Attempt to integrate; increase a power by 1 Two terms correct All correct (ignore $+c$ even outside [ ]) Attempt to evaluate limits at −1 and 2 Penalise if $+c$ remains $\left(\dfrac{189}{4}\right)$
(ii)	Area of triangle $= \dfrac{1}{2} \times 3 \times 18 = 27$ Shaded area = (b)(i) ans – triangle $= 20.25$ oe	B1 M1 A1	 3	oe $47.25 - 27$ $\dfrac{81}{4}$
(c)	$f(-1.1) = 0.509$ (or -2.491) } (both) $f(-1.2) = -2.168$ (or -5.168) Change of sign $\Rightarrow$ root in interval $(-1.2, -1.1)$	M1 A1	 2	both $f(x) = x^3 - 6x^2 + 9x + 19$ (or 16 or 13) May consider $g(x) > -3$ and $g(x) < -3$ Must have correct values or use $f(-1.1) > 0$, $f(-1.2) < 0$ with full explanation to score A1
	Total		**16**	

The function f is defined for all values of x by

$$f(x) = x^3 - 7x^2 + 14x - 8.$$

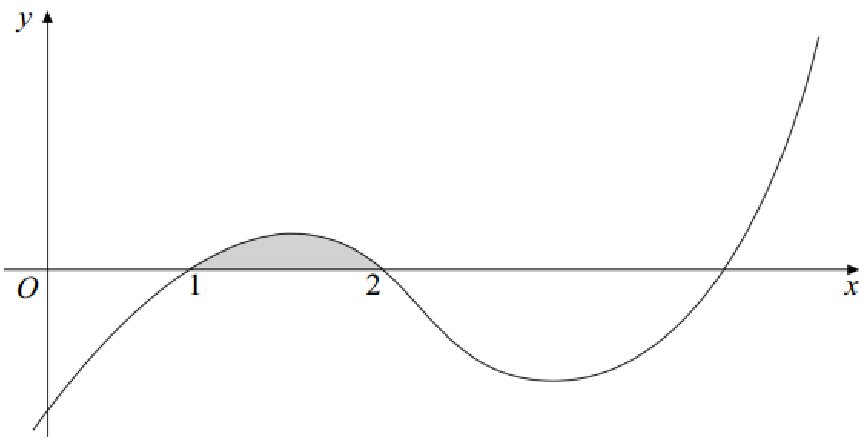
It is given that $f(1) = 0$ and $f(2) = 0$.

(a) Find the values of $f(3)$ and $f(4)$. *(2 marks)*

(b) Write $f(x)$ as a product of **three** linear factors. *(2 marks)*

(c) The diagram shows the graph of

$$y = x^3 - 7x^2 + 14x - 8.$$



(i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) State, giving a reason, whether the function f is increasing or decreasing at the point where $x = 3$. *(2 marks)*

(iii) Find $\displaystyle\int (x^3 - 7x^2 + 14x - 8)\,dx$. *(3 marks)*

(iv) Hence find the area of the shaded region enclosed by the graph of $y = f(x)$, for $1 \leqslant x \leqslant 2$, and the x-axis. *(3 marks)*

Q	Solution	Marks	Total	Comments
8 (a)	$\mathrm{f}(3) = -2,\ \mathrm{f}(4) = 0$	B1B1	2	
(b)	Awareness of factor theorem	M1		PI by answers involving 1, 2, 4
	$\mathrm{f}(x) = (x-1)(x-2)(x-4)$	A1	2	M1A0 for $(x+1)(x+2)(x+4)$ or for two factors correct
(c)(i)	$y' = 3x^2 - 14x + 14$	B3	3	B1 for each term
(ii)	Gradient at $x = 3$ is -1	B1F		ft one wrong coefficient
	Function is decreasing	E1F	2	ft wrong (non-zero) value for gradient at $x = 3$ Alternative methods: 2/2 for convincing argument based on SP at $x \approx 3.22$ or values $\mathrm{f}(a), \mathrm{f}(b)$ where $a \le 3 < b$
(iii)	$\int y\,\mathrm{d}x = \frac{1}{4}x^4 - \frac{7}{3}x^3 + 7x^2 - 8x(+c)$	M1A2	3	M1 if at least one term correct; -1 EE
(iv)	Substitution of $x = 1$ and/or $x = 2$	M1		in c's integral (not y or y')
	Both substitutions and subtraction	m1		Subtraction must be right way round
	Area = $\frac{5}{12}$	A1	3	allow AWRT 0.416 or 0.417
	Total		**15**	