Exponentials and Logarithms

Given that $p = \log_2 3$ and $q = \log_2 5$, find expressions in terms of p and q for

(a) $\log_2 45$, **(3)**

(b) $\log_2 0.3$ **(3)**

(a) $= \log_2 (3^2 \times 5)$ B1
$= 2\log_2 3 + \log_2 5 = 2p + q$ M1 A1

(b) $= \log_2 \frac{3}{5\times 2} = \log_2 3 - \log_2 5 - \log_2 2$ M1
$= p - q - 1$ B1 A1 **(6)**

(a) Given that $t = \log_3 x$, find expressions in terms of t for

(i) $\log_3 x^2$,

(ii) $\log_9 x$. **(4)**

(b) Hence, or otherwise, find to 3 significant figures the value of x such that

$$\log_3 x^2 - \log_9 x = 4.$$ **(3)**

(a) *(i)* $= 2\log_3 x = 2t$ M1 A1

(ii) $= \frac{\log_3 x}{\log_3 9} = \frac{\log_3 x}{2} = \frac{1}{2}t$ M1 A1

(b) $2t - \frac{1}{2}t = 4$

$t = \frac{8}{3}$ M1

$\log_3 x = \frac{8}{3}$, $x = 3^{\frac{8}{3}} = 18.7$ M1 A1 **(7)**

(a) Evaluate

$$\log_3 27 - \log_8 4.$$ **(4)**

(b) Solve the equation

$$4^x - 3(2^{x+1}) = 0.$$ **(5)**

(a) $= 3 - \log_8 8^{\frac{2}{3}}$ B1 M1 A1

$= 3 - \frac{2}{3} = \frac{7}{3}$ A1

(b) $(2^2)^x - 3(2 \times 2^x) = 0$ M1

$(2^x)^2 - 6(2^x) = 0$

$2^x(2^x - 6) = 0$ M1

$2^x = 0$ (no solutions) or 6 A1

$x = \frac{\lg 6}{\lg 2} = 2.58$ (3sf) M1 A1 **(9)**

(a) Given that

$$\log_2 (y - 1) = 1 + \log_2 x,$$

show that

$$y = 2x + 1.$$ **(3)**

(b) Solve the simultaneous equations

$$\log_2 (y - 1) = 1 + \log_2 x$$

$$2 \log_3 y = 2 + \log_3 x$$ **(7)**

(a) $\log_2 (y - 1) - \log_2 x = 1, \quad \log_2 \frac{y-1}{x} = 1$ M1

$\frac{y-1}{x} = 2^1 = 2$ M1

$y - 1 = 2x, \quad y = 2x + 1$ A1

(b) $2 \log_3 y = 2 + \log_3 x \Rightarrow \log_3 y^2 - \log_3 x = 2$ M1

$\frac{y^2}{x} = 3^2 = 9$ M1

$y^2 = 9x$ A1

sub. $y = 2x + 1$ $(2x + 1)^2 = 9x$ M1

$4x^2 + 4x + 1 = 9x$

$4x^2 - 5x + 1 = 0$

$(4x - 1)(x - 1) = 0$ M1

$x = \frac{1}{4}, 1$ A1

$\therefore \ x = \frac{1}{4}, y = \frac{3}{2}$ or $x = 1, y = 3$ A1 **(10)**