# Core 4: Trigonometry

Past Exam Questions
2006 - 2013

Name:

January 2006

**3** It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of $R$. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of $\theta$ at which this maximum value occurs. *(3 marks)*

**6** (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where $a$ and $b$ are constants. *(2 marks)*

June 2006

**4** (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*

(b) Show that

$$\sin 2x - \tan x = \tan x \cos 2x$$

for all values of $x$. *(3 marks)*

(c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0° < x < 360°$. *(4 marks)*

January 2007

**3** (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of $x$. *(2 marks)*

(ii) Solve the equation $3\sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

**7** (a) Use the identity

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$. *(2 marks)*

(b) Show that

$$2 - 2\tan x - \frac{2\tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of $x$, $\tan 2x \neq 0$. *(4 marks)*

June 2007

**3** (a) Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving your value for $\alpha$ to the nearest $0.1°$. *(3 marks)*

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest $0.1°$. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of $x$ in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

January 2008

**7** (a) (i) Express $6\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value for $\alpha$ to the nearest $0.1°$. *(2 marks)*

(ii) Hence solve the equation $6\sin 2x + 8\cos 2x = 7$, giving all solutions to the nearest $0.1°$ in the interval $0° < x < 360°$. *(4 marks)*

(b) (i) Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \dfrac{1}{\tan x}$. *(4 marks)*

(ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval $0° < x < 360°$. *(4 marks)*

June 2008

**3** (a) By writing $\sin 3x$ as $\sin(x + 2x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$ for all values of $x$. *(5 marks)*

**5** (a) The angle $\alpha$ is acute and $\sin\alpha = \frac{4}{5}$.

(i) Find the value of $\cos\alpha$. *(1 mark)*

(ii) Express $\cos(\alpha - \beta)$ in terms of $\sin\beta$ and $\cos\beta$. *(2 marks)*

(iii) Given also that the angle $\beta$ is acute and $\cos\beta = \frac{5}{13}$, find the exact value of $\cos(\alpha - \beta)$. *(2 marks)*

(b) (i) Given that $\tan 2x = 1$, show that $\tan^2 x + 2\tan x - 1 = 0$. *(2 marks)*

(ii) Hence, given that $\tan 45° = 1$, show that $\tan 22\frac{1}{2}° = \sqrt{2} - 1$. *(3 marks)*

January 2009

**2** (a) Express $\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of $\alpha$ in radians to two decimal places. *(3 marks)*

(b) Hence:

(i) write down the minimum value of $\sin x - 3\cos x$; *(1 mark)*

(ii) find the value of $x$ in the interval $0 < x < 2\pi$ at which this minimum value occurs, giving your value of $x$ in radians to two decimal places. *(2 marks)*

**5** (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*

(b) Solve the equation

$$5\sin 2x + 3\cos x = 0$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$ to the nearest 0.1°, where appropriate. *(4 marks)*

(c) Given that $\sin 2x + \cos 2x = 1 + \sin x$ and $\sin x \neq 0$, show that $2(\cos x - \sin x) = 1$. *(4 marks)*

June 2009

**6** (a) (i) Show that the equation $3\cos 2x + 7\cos x + 5 = 0$ can be written in the form $a\cos^2 x + b\cos x + c = 0$, where $a$, $b$ and $c$ are integers. *(3 marks)*

(ii) Hence find the possible values of $\cos x$. *(2 marks)*

(b) (i) Express $7\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $\alpha$ is an acute angle. Give your value of $\alpha$ to the nearest 0.1°. *(3 marks)*

(ii) Hence solve the equation $7\sin\theta + 3\cos\theta = 4$ for all solutions in the interval $0° \leqslant \theta \leqslant 360°$, giving $\theta$ to the nearest 0.1°. *(3 marks)*

(c) (i) Given that $\beta$ is an acute angle and that $\tan\beta = 2\sqrt{2}$, show that $\cos\beta = \frac{1}{3}$. *(2 marks)*

(ii) Hence show that $\sin 2\beta = p\sqrt{2}$, where $p$ is a rational number. *(2 marks)*

January 2010

**2** (a) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of $\alpha$, in radians, to three decimal places. *(3 marks)*

(b) (i) Hence write down the minimum value of $\cos x + 3\sin x$. *(1 mark)*

(ii) Find the value of $x$ in the interval $0 \leqslant x \leqslant 2\pi$ at which this minimum occurs, giving your answer, in radians, to three decimal places. *(2 marks)*

(c) Solve the equation $\cos x + 3\sin x = 2$ in the interval $0 \leqslant x \leqslant 2\pi$, giving all solutions, in radians, to three decimal places. *(4 marks)*

**6** (a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. *(2 marks)*

(ii) Given that $0 < \theta < \frac{\pi}{2}$ and $\cos\theta = \frac{3}{5}$, show that $\sin 2\theta = \frac{24}{25}$ and find the value of $\cos 2\theta$. *(2 marks)*

June 2010

**5 (a) (i)** Show that the equation $3\cos 2x + 2\sin x + 1 = 0$ can be written in the form

$$3\sin^2 x - \sin x - 2 = 0$$

*(3 marks)*

**(ii)** Hence, given that $3\cos 2x + 2\sin x + 1 = 0$, find the possible values of $\sin x$. *(2 marks)*

**(b) (i)** Express $3\cos 2x + 2\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving $\alpha$ to the nearest 0.1°. *(3 marks)*

**(ii)** Hence solve the equation

$$3\cos 2x + 2\sin 2x + 1 = 0$$

for all solutions in the interval $0° < x < 180°$, giving $x$ to the nearest 0.1°. *(3 marks)*

January 2011

**1 (a)** Express $2\sin x + 5\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your value of $\alpha$ to the nearest 0.1°. *(3 marks)*

**(b) (i)** Write down the maximum value of $2\sin x + 5\cos x$. *(1 mark)*

**(ii)** Find the value of $x$ in the interval $0° \leqslant x \leqslant 360°$ at which this maximum occurs, giving the value of $x$ to the nearest 0.1°. *(2 marks)*

**6 (a) (i)** Given that $\tan 2x + \tan x = 0$, show that $\tan x = 0$ or $\tan^2 x = 3$. *(3 marks)*

**(ii)** Hence find all solutions of $\tan 2x + \tan x = 0$ in the interval $0° < x < 180°$. *(1 mark)*

**(b) (i)** Given that $\cos x \neq 0$, show that the equation

$$\sin 2x = \cos x \cos 2x$$

can be written in the form

$$2\sin^2 x + 2\sin x - 1 = 0$$ *(3 marks)*

**(ii)** Show that all solutions of the equation $2\sin^2 x + 2\sin x - 1 = 0$ are given by $\sin x = \dfrac{\sqrt{3} - 1}{p}$, where $p$ is an integer. *(3 marks)*

January 2012

**2** Angle $\alpha$ is acute and $\cos\alpha = \frac{3}{5}$. Angle $\beta$ is **obtuse** and $\sin\beta = \frac{1}{2}$.

**(a) (i)** Find the value of $\tan\alpha$ as a fraction. *(1 mark)*

**(ii)** Find the value of $\tan\beta$ in surd form. *(2 marks)*

**(b)** Hence show that $\tan(\alpha + \beta) = \dfrac{m\sqrt{3} - n}{n\sqrt{3} + m}$, where $m$ and $n$ are integers. *(3 marks)*

**6 (a)** Use the Factor Theorem to show that $4x - 3$ is a factor of

$$16x^3 + 11x - 15$$ *(2 marks)*

**(b)** Given that $x = \cos\theta$, show that the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

can be written in the form

$$16x^3 + 11x - 15 = 0$$ *(4 marks)*

**(c)** Hence show that the only solutions of the equation

$$27\cos\theta\cos 2\theta + 19\sin\theta\sin 2\theta - 15 = 0$$

are given by $\cos\theta = \frac{3}{4}$. *(4 marks)*

June 2012

**2 (a)** Express $\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R>0$ and $0° < \alpha < 90°$, giving your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

**(b)** Hence find the values of $x$ in the interval $0° < x < 360°$ for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of $x$ to the nearest degree. *(4 marks)*

January 2013

**3 (a) (i)** Express $3\cos x + 2\sin x$ in the form $R\cos(x-\alpha)$, where $R>0$ and $0° < \alpha < 90°$, giving your value of $\alpha$ to the nearest $0.1°$. *(3 marks)*

**(ii)** Hence find the minimum value of $3\cos x + 2\sin x$ and the value of $x$ in the interval $0° < x < 360°$ where the minimum occurs. Give your value of $x$ to the nearest $0.1°$. *(3 marks)*

**(b) (i)** Show that $\cot x - \sin 2x = \cot x \cos 2x$ for $0° < x < 180°$. *(3 marks)*

**(ii)** Hence, or otherwise, solve the equation

$$\cot x - \sin 2x = 0$$

in the interval $0° < x < 180°$. *(3 marks)*

June 2013

**2** The acute angles $\alpha$ and $\beta$ are given by $\tan\alpha = \frac{2}{\sqrt{5}}$ and $\tan\beta = \frac{1}{2}$.

**(a) (i)** Show that $\sin\alpha = \frac{2}{3}$, and find the exact value of $\cos\alpha$. *(2 marks)*

**(ii)** Hence find the exact value of $\sin 2\alpha$. *(2 marks)*

**(b)** Show that the exact value of $\cos(\alpha - \beta)$ can be expressed as $\frac{2}{15}(k+\sqrt{5})$, where $k$ is an integer. *(4 marks)*

**5 (c) (i)** Show that the equation $2\cos 2\theta \sin\theta + 9\sin\theta + 3 = 0$ can be written as $4x^3 - 11x - 3 = 0$, where $x = \sin\theta$. *(3 marks)*