FP1: Summations

Past Exam Questions
2006 - 2013

Name:

Summations

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

June 2006

3 Show that

$$\sum_{r=1}^{n}(r^2 - r) = kn(n+1)(n-1)$$

where k is a rational number. *(4 marks)*

January 2007

6 (a) (i) Expand $(2r-1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n}(2r-1)^2 = \tfrac{1}{3}n(4n^2-1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

January 2008

4 (a) Find

$$\sum_{r=1}^{n}(r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

$$S = \sum_{r=1}^{1000}(r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

January 2009

4 It is given that

$$S_n = \sum_{r=1}^{n} (3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

January 2010

8 (a) Show that

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a, b and c are integers. *(4 marks)*

(b) Show that there is exactly one positive integer n for which

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2$$ *(5 marks)*

January 2011

8 Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

January 2012

4 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where k is a constant. *(5 marks)*

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$ *(2 marks)*

January 2013

8 (a) Show that

$$\sum_{r=1}^{n} 2r(2r^2-3r-1) = n(n+p)(n+q)^2$$

where p and q are integers to be found. *(6 marks)*

(b) Hence find the value of

$$\sum_{r=11}^{20} 2r(2r^2-3r-1)$$ *(2 marks)*

June 2013

7 (b) It is given that $S_n = \sum_{r=1}^{n} (2r-1)^2$.

(i) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = \frac{n}{3}(kn^2-1)$, where k is an integer to be found. *(5 marks)*

(ii) Hence show that $6S_n$ can be written as the product of three consecutive integers. *(2 marks)*

(c) Find the smallest value of N for which the sum of the squares of the first N odd numbers is greater than $180\,000$. *(2 marks)*