S1: Probability

Past Paper Questions
2006 - 2013

Name:

January 2006

2 Xavier, Yuri and Zara attend a sports centre for their judo club's practice sessions. The probabilities of them arriving late are, independently, 0.3, 0.4 and 0.2 respectively.

(a) Calculate the probability that for a particular practice session:

(i) all three arrive late; *(1 mark)*

(ii) none of the three arrives late; *(2 marks)*

(iii) only Zara arrives late. *(2 marks)*

(b) Zara's friend, Wei, also attends the club's practice sessions. The probability that Wei arrives late is 0.9 when Zara arrives late, and is 0.25 when Zara does not arrive late.

Calculate the probability that for a particular practice session:

(i) both Zara and Wei arrive late; *(2 marks)*

(ii) either Zara or Wei, but not both, arrives late. *(3 marks)*

June 2006

6 A housing estate consists of 320 houses: 120 detached and 200 semi-detached. The numbers of children living in these houses are shown in the table.

	Number of children				
	None	**One**	**Two**	**At least three**	**Total**
Detached house	24	32	41	23	120
Semi-detached house	40	37	88	35	200
Total	64	69	129	58	320

A house on the estate is selected at random.

D denotes the event 'the house is detached'.
R denotes the event 'no children live in the house'.
S denotes the event 'one child lives in the house'.
T denotes the event 'two children live in the house'.
(D' denotes the event 'not D'.)

(a) Find:

(i) $\mathrm{P}(D)$; *(1 mark)*

(ii) $\mathrm{P}(D \cap R)$; *(1 mark)*

(iii) $\mathrm{P}(D \cup T)$; *(2 marks)*

(iv) $\mathrm{P}(D \mid R)$; *(2 marks)*

(v) $\mathrm{P}(R \mid D')$. *(3 marks)*

(b) (i) Name two of the events D, R, S and T that are mutually exclusive. *(1 mark)*

(ii) Determine whether the events D and R are independent. Justify your answer. *(2 marks)*

(c) Define, in the context of this question, the event:

(i) $D' \cup T$; *(2 marks)*

(ii) $D \cap (R \cup S)$. *(2 marks)*

January 2007

5 Dafydd, Eli and Fabio are members of an amateur cycling club that holds a time trial each Sunday during the summer. The independent probabilities that Dafydd, Eli and Fabio take part in any one of these trials are 0.6, 0.7 and 0.8 respectively.

Find the probability that, on a particular Sunday during the summer:

(a) none of the three cyclists takes part; *(2 marks)*

(b) Fabio is the only one of the three cyclists to take part; *(2 marks)*

(c) exactly one of the three cyclists takes part; *(3 marks)*

(d) either one or two of the three cyclists take part. *(3 marks)*

June 2007

2 The British and Irish Lions 2005 rugby squad contained 50 players. The nationalities and playing positions of these players are shown in the table.

		Nationality			
		English	**Welsh**	**Scottish**	**Irish**
Playing position	**Forward**	14	5	2	6
	Back	8	7	2	6

(a) A player was selected at random from the squad for a radio interview. Calculate the probability that the player was:

(i) a Welsh back; *(1 mark)*

(ii) English; *(2 marks)*

(iii) not English; *(1 mark)*

(iv) Irish, given that the player was a back; *(2 marks)*

(v) a forward, given that the player was not Scottish. *(2 marks)*

(b) Four players were selected at random from the squad to visit a school. Calculate the probability that all four players were English. *(3 marks)*

January 2008

5 A health club has a number of facilities which include a gym and a sauna. Andrew and his wife, Heidi, visit the health club together on Tuesday evenings.

On any visit, Andrew uses either the gym or the sauna or both, but no other facilities. The probability that he uses the gym, $P(G)$, is 0.70. The probability that he uses the sauna, $P(S)$, is 0.55. The probability that he uses both the gym and the sauna is 0.25.

(a) Calculate the probability that, on a particular visit:

(i) he does not use the gym; *(1 mark)*

(ii) he uses the gym but not the sauna; *(2 marks)*

(iii) he uses either the gym or the sauna but not both. *(2 marks)*

(b) Assuming that Andrew's decision on what facility to use is independent from visit to visit, calculate the probability that, during a month in which there are exactly four Tuesdays, he does not use the gym. *(2 marks)*

(c) The probability that Heidi uses the gym when Andrew uses the gym is 0.6, but is only 0.1 when he does not use the gym.

Calculate the probability that, on a particular visit, Heidi uses the gym. *(3 marks)*

(d) On any visit, Heidi uses **exactly one** of the club's facilities.

The probability that she uses the sauna is 0.35.

Calculate the probability that, on a particular visit, she uses a facility other than the gym or the sauna. *(2 marks)*

June 2008

2 A basket in a stationery store contains a total of 400 marker and highlighter pens. Of the marker pens, some are permanent and the rest are non-permanent. The colours and types of pen are shown in the table.

	Colour			
Type	**Black**	**Blue**	**Red**	**Green**
Permanent marker	44	66	32	18
Non-permanent marker	36	53	21	10
Highlighter	0	41	37	42

A pen is selected at random from the basket. Calculate the probability that it is:

(a) a blue pen; *(1 mark)*

(b) a marker pen; *(2 marks)*

(c) a blue pen or a marker pen; *(2 marks)*

(d) a green pen, given that it is a highlighter pen; *(2 marks)*

(e) a non-permanent marker pen, given that it is a red pen. *(2 marks)*

January 2009

4 Gary and his neighbour Larry work at the same place.

On any day when Gary travels to work, he uses one of three options: his car only, a bus only or both his car and a bus. The probability that he uses his car, either on its own or with a bus, is 0.6. The probability that he uses both his car and a bus is 0.25.

(a) Calculate the probability that, on any particular day when Gary travels to work, he:

(i) does not use his car; *(1 mark)*

(ii) uses his car only; *(2 marks)*

(iii) uses a bus. *(3 marks)*

(b) On any day, the probability that Larry travels to work with Gary is 0.9 when Gary uses his car only, is 0.7 when Gary uses both his car and a bus, and is 0.3 when Gary uses a bus only.

(i) Calculate the probability that, on any particular day when Gary travels to work, Larry travels with him. *(4 marks)*

(ii) Assuming that option choices are independent from day to day, calculate, to three decimal places, the probability that, during any particular week (5 days) when Gary travels to work every day, Larry never travels with him. *(2 marks)*

June 2009

1 A large bookcase contains two types of book: hardback and paperback. The number of books of each type in each of four subject categories is shown in the table.

		Subject category				
		Crime	**Romance**	**Science fiction**	**Thriller**	**Total**
Type	**Hardback**	8	16	18	18	60
	Paperback	16	40	14	30	100
	Total	24	56	32	48	160

(a) A book is selected at random from the bookcase. Calculate the probability that the book is:

(i) a paperback; *(1 mark)*

(ii) not science fiction; *(2 marks)*

(iii) science fiction or a hardback; *(2 marks)*

(iv) a thriller, given that it is a paperback. *(2 marks)*

(b) Three books are selected at random, without replacement, from the bookcase.

Calculate, to three decimal places, the probability that one is crime, one is romance and one is science fiction. *(4 marks)*

4 Each school-day morning, three students, Rita, Said and Ting, travel independently from their homes to the same school by one of three methods: walk, cycle or bus. The table shows the probabilities of their independent daily choices.

	Walk	**Cycle**	**Bus**
Rita	0.65	0.10	0.25
Said	0.40	0.45	0.15
Ting	0.25	0.55	0.20

(a) Calculate the probability that, on any given school-day morning:

(i) all 3 students walk to school; *(2 marks)*

(ii) only Rita travels by bus to school; *(2 marks)*

(iii) at least 2 of the 3 students cycle to school. *(4 marks)*

(b) Ursula, a friend of Rita, never travels to school by bus. The probability that:

Ursula walks to school when Rita walks to school is 0.9;
Ursula cycles to school when Rita cycles to school is 0.7.

Calculate the probability that, on any given school-day morning, Rita and Ursula travel to school by:

(i) the same method; *(3 marks)*

(ii) different methods. *(1 mark)*

5 Hugh owns a small farm.

(a) He has two sows, Josie and Rosie, which he feeds at a trough in their field at 8.00 am each day.

The probability that Josie is waiting at the trough at 8.00 am on any given day is 0.90. The probability that Rosie is waiting at the trough at 8.00 am on any given day is 0.70 when Josie is waiting at the trough, but is only 0.20 when Josie is not waiting at the trough.

Calculate the probability that, at 8.00 am on a given day:

(i) both sows are waiting at the trough; *(2 marks)*

(ii) neither sow is waiting at the trough; *(2 marks)*

(iii) at least one sow is waiting at the trough. *(1 mark)*

(b) Hugh also has two cows, Daisy and Maisy. Each day at 4.00 pm, he collects them from the gate to their field and takes them to be milked.

The probability, P(D), that Daisy is waiting at the gate at 4.00 pm on any given day is 0.75.
The probability, P(M), that Maisy is waiting at the gate at 4.00 pm on any given day is 0.60.
The probability that both Daisy and Maisy are waiting at the gate at 4.00 pm on any given day is 0.40.

(i) In the table of probabilities, D' and M' denote the events 'not D' and 'not M' respectively.

	M	M'	Total
D	0.40		0.75
D'			
Total	0.60		1.00

Complete the copy of this table which is printed on page 13. *(2 marks)*

(ii) Hence, or otherwise, find the probability that, at 4.00 pm on a given day:

(A) neither cow is waiting at the gate; *(1 mark)*

(B) only Daisy is waiting at the gate; *(1 mark)*

(C) exactly one cow is waiting at the gate. *(2 marks)*

2 The number of MPs in the House of Commons was 645 at the beginning of August 2009. The genders of these MPs and the political parties to which they belonged are shown in the table.

		Political Party				
		Labour	**Conservative**	**Liberal Democrat**	**Other**	**Total**
Gender	**Male**	255	175	54	35	519
	Female	94	18	9	5	126
	Total	349	193	63	40	645

(a) One MP was selected at random for an interview. Calculate, to three decimal places, the probability that the MP was:

(i) a male Conservative; *(1 mark)*

(ii) a male; *(1 mark)*

(iii) a Liberal Democrat; *(1 mark)*

(iv) Labour, given that the MP was female; *(2 marks)*

(v) male, given that the MP was **not** Labour. *(3 marks)*

(b) Two **female** MPs were selected at random for an enquiry. Calculate, to three decimal places, the probability that both MPs were Labour. *(2 marks)*

(c) Three MPs were selected at random for a committee. Calculate, to three decimal places, the probability that exactly one MP was Labour and exactly one MP was Conservative. *(4 marks)*

5 (a) Emma visits her local supermarket every Thursday to do her weekly shopping.

The event that she buys orange juice is denoted by J, and the event that she buys bottled water is denoted by W. At each visit, Emma may buy neither, or one, or both of these items.

(i) Complete the table of probabilities, printed below, for these events, where J' and W' denote the events 'not J' and 'not W' respectively. *(3 marks)*

(ii) Hence, or otherwise, find the probability that, on any given Thursday, Emma buys either orange juice or bottled water but not both. *(2 marks)*

(iii) Show that:

(A) the events J and W are **not** mutually exclusive;

(B) the events J and W are **not** independent. *(3 marks)*

(b) Rhys visits the supermarket every Saturday to do his weekly shopping. Items that he may buy are milk, cheese and yogurt.

The probability, $P(M)$, that he buys milk on any given Saturday is 0.85.
The probability, $P(C)$, that he buys cheese on any given Saturday is 0.60.
The probability, $P(Y)$, that he buys yogurt on any given Saturday is 0.55.
The events M, C and Y may be assumed to be independent.

Calculate the probability that, on any given Saturday, Rhys buys:

(i) none of the 3 items; *(2 marks)*

(ii) exactly 2 of the 3 items. *(3 marks)*

	J	J'	Total
W			0.65
W'	0.15		
Total		0.30	1.00

6 Twins Alec and Eric are members of the same local cricket club and play for the club's under 18 team.

The probability that Alec is selected to play in any particular game is 0.85.
The probability that Eric is selected to play in any particular game is 0.60.
The probability that both Alec and Eric are selected to play in any particular game is 0.55.

(a) By using a table, or otherwise:

(i) show that the probability that neither twin is selected for a particular game is 0.10;

(ii) find the probability that at least one of the twins is selected for a particular game;

(iii) find the probability that exactly one of the twins is selected for a particular game.

(5 marks)

(b) The probability that the twins' younger brother, Cedric, is selected for a particular game is:

0.30 given that both of the twins have been selected;
0.75 given that exactly one of the twins has been selected;
0.40 given that neither of the twins has been selected.

Calculate the probability that, for a particular game:

(i) all three brothers are selected;

(ii) at least two of the three brothers are selected. *(6 marks)*

4 A survey of the 640 properties on an estate was undertaken. Part of the information collected related to the number of bedrooms and the number of toilets in each property.

This information is shown in the table.

		Number of toilets				
		1	**2**	**3**	**4 or more**	**Total**
Number of bedrooms	**1**	46	14	0	0	60
	2	24	67	23	0	114
	3	7	72	99	16	194
	4	0	19	123	48	190
	5 or more	0	0	11	71	82
	Total	77	172	256	135	640

(a) A property on the estate is selected at random.

Find, giving your answer to three decimal places, the probability that the property has:

(i) exactly 3 bedrooms; *(1 mark)*

(ii) at least 2 toilets; *(2 marks)*

(iii) exactly 3 bedrooms and at least 2 toilets; *(2 marks)*

(iv) at most 3 bedrooms, given that it has exactly 2 toilets. *(3 marks)*

(b) Use relevant answers from part **(a)** to show that the number of toilets is **not** independent of the number of bedrooms. *(2 marks)*

(c) Three properties are selected at random from those on the estate which have exactly 3 bedrooms.

Calculate the probability that one property has 2 toilets, one has 3 toilets and the other has at least 4 toilets. Give your answer to three decimal places. *(4 marks)*

January 2013

5 Roger is an active retired lecturer. Each day after breakfast, he decides whether the weather for that day is going to be fine (F), dull (D) or wet (W). He then decides on only one of four activities for the day: cycling (C), gardening (G), shopping (S) or relaxing (R). His decisions from day to day may be assumed to be independent.

The table shows Roger's probabilities for each combination of weather and activity.

		Weather		
		Fine (F)	**Dull (D)**	**Wet (W)**
Activity	**Cycling (C)**	0.30	0.10	0
	Gardening (G)	0.25	0.05	0
	Shopping (S)	0	0.10	0.05
	Relaxing (R)	0	0.05	0.10

(a) Find the probability that, on a particular day, Roger decided:

(i) that it was going to be fine and that he would go cycling;

(ii) on either gardening or shopping;

(iii) to go cycling, given that he had decided that it was going to be fine;

(iv) **not** to relax, given that he had decided that it was going to be dull;

(v) that it was going to be fine, given that he did **not** go cycling. *(9 marks)*

(b) Calculate the probability that, on a particular Saturday and Sunday, Roger decided that it was going to be fine and decided on the same activity for both days. *(3 marks)*

June 2013

5 Alison is a member of a tenpin bowling club which meets at a bowling alley on Wednesday and Thursday evenings.

The probability that she bowls on a Wednesday evening is 0.90. Independently, the probability that she bowls on a Thursday evening is 0.95.

(a) Calculate the probability that, during a particular week, Alison bowls on:

(i) two evenings;

(ii) exactly one evening. *(3 marks)*

(b) David, a friend of Alison, is a member of the same club.

The probability that he bowls on a Wednesday evening, given that Alison bowls on that evening, is 0.80. The probability that he bowls on a Wednesday evening, given that Alison does not bowl on that evening, is 0.15.

The probability that he bowls on a Thursday evening, given that Alison bowls on that evening, is 1. The probability that he bowls on a Thursday evening, given that Alison does not bowl on that evening, is 0.

Calculate the probability that, during a particular week:

(i) Alison and David bowl on a Wednesday evening; *(2 marks)*

(ii) Alison and David bowl on both evenings; *(2 marks)*

(iii) Alison, but not David, bowls on a Thursday evening; *(1 mark)*

(iv) neither bowls on either evening. *(3 marks)*