S1: Confidence Intervals

Past Paper Questions
2006 - 2013

Name:

January 2006

3 When an alarm is raised at a market town's fire station, the fire engine cannot leave until at least five fire-fighters arrive at the station. The call-out time, X minutes, is the time between an alarm being raised and the fire engine leaving the station.

The value of X was recorded on a random sample of 50 occasions. The results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 286.5 \qquad \sum (x - \bar{x})^2 = 45.16$$

(a) Find values for the mean and standard deviation of this sample of 50 call-out times. *(2 marks)*

(b) Hence construct a 99% confidence interval for the mean call-out time. *(4 marks)*

(c) The fire and rescue service claims that the station's mean call-out time is less than 5 minutes, whereas a parish councillor suggests that it is more than $6\frac{1}{2}$ minutes.

Comment on **each** of these claims. *(2 marks)*

June 2006

4 The weights of packets of sultanas may be assumed to be normally distributed with a standard deviation of 6 grams.

The weights of a random sample of 10 packets were as follows:

498 496 499 511 503 505 510 509 513 508

(a) (i) Construct a 99% confidence interval for the mean weight of packets of sultanas, giving the limits to one decimal place. *(5 marks)*

(ii) State why, in calculating your confidence interval, use of the Central Limit Theorem was **not** necessary. *(1 mark)*

(iii) On each packet it states 'Contents 500 grams'.

Comment on this statement using **both** the given sample **and** your confidence interval. *(3 marks)*

(b) Given that the mean weight of all packets of sultanas is 500 grams, state the probability that a 99% confidence interval for the mean, calculated from a random sample of packets, will **not** contain 500 grams. *(1 mark)*

January 2007

4 A very popular play has been performed at a London theatre on each of 6 evenings per week for about a year. Over the past 13 weeks (78 performances), records have been kept of the proceeds from the sales of programmes at each performance. An analysis of these records has found that the mean was £184 and the standard deviation was £32.

(a) Assuming that the 78 performances may be considered to be a random sample, construct a 90% confidence interval for the mean proceeds from the sales of programmes at an evening performance of this play. *(4 marks)*

(b) Comment on the likely validity of the assumption in part (a) when constructing a confidence interval for the mean proceeds from the sales of programmes at an evening performance of:

(i) this particular play;

(ii) any play. *(3 marks)*

June 2007

3 (a) A sample of 50 washed baking potatoes was selected at random from a large batch. The weights of the 50 potatoes were found to have a mean of 234 grams and a standard deviation of 25.1 grams.

Construct a 95% confidence interval for the mean weight of potatoes in the batch. *(4 marks)*

(b) The batch of potatoes is purchased by a market stallholder. He sells them to his customers by allowing them to choose any 5 potatoes for £1.

Give a reason why such chosen potatoes are unlikely to represent a random sample from the batch. *(1 mark)*

January 2008

3 The height, in metres, of adult male African elephants may be assumed to be normally distributed with mean μ and standard deviation 0.20.

The heights of a sample of 12 such elephants were measured with the following results, in metres.

3.37 3.45 2.93 3.42 3.49 3.67 2.96 3.57 3.36 2.89 3.22 2.91

(a) Stating a necessary assumption, construct a 98% confidence interval for μ. *(6 marks)*

(b) The mean height of adult male **Asian** elephants is known to be 2.90 metres.

Using your confidence interval, state, with a reason, what can be concluded about the mean heights of adult males in these two types of elephant. *(2 marks)*

7 Vernon, a service engineer, is expected to carry out a boiler service in one hour.

One hour is subtracted from each of his actual times, and the resulting differences, x minutes, for a random sample of 100 boiler services are summarised in the table.

Difference	**Frequency**
$-6 \leqslant x < -4$	4
$-4 \leqslant x < -2$	9
$-2 \leqslant x < 0$	13
$0 \leqslant x < 2$	27
$2 \leqslant x < 4$	21
$4 \leqslant x < 6$	15
$6 \leqslant x < 8$	7
$8 \leqslant x \leqslant 10$	4
Total	**100**

(a) (i) Calculate estimates of the mean and the standard deviation of these differences. *(4 marks)*

(ii) Hence deduce, in minutes, estimates of the mean and the standard deviation of Vernon's actual service times for this sample. *(3 marks)*

(b) (i) Construct an approximate 98% confidence interval for the mean time taken by Vernon to carry out a boiler service. *(4 marks)*

(ii) Give a reason why this confidence interval is approximate rather than exact. *(1 mark)*

(c) Vernon claims that, more often than not, a boiler service takes more than an hour and that, on average, a boiler service takes much longer than an hour.

Comment, with a justification, on **each** of these claims. *(2 marks)*

January 2009

5 The times taken by new recruits to complete an assault course may be modelled by a normal distribution with a standard deviation of 8 minutes.

A group of 30 new recruits takes a total time of 1620 minutes to complete the course.

(a) Calculate the mean time taken by these 30 new recruits. *(1 mark)*

(b) Assuming that the 30 recruits may be considered to be a random sample, construct a 98% confidence interval for the mean time taken by new recruits to complete the course. *(4 marks)*

(c) Construct an interval within which approximately 98% of the times taken by individual new recruits to complete the course will lie. *(2 marks)*

(d) State where, if at all, in this question you made use of the Central Limit Theorem. *(1 mark)*

June 2009

6 (a) The time taken, in minutes, by *Domesat* to install a domestic satellite system may be modelled by a normal distribution with unknown mean, μ, and standard deviation 15.

The times taken, in minutes, for a random sample of 10 installations are as follows.

47 39 25 51 47 36 63 41 78 43

Construct a 98% confidence interval for μ. *(5 marks)*

(b) The time taken, Y minutes, by *Teleair* to erect a TV aerial and then connect it to a TV is known to have a mean of 108 and a standard deviation of 28.

Estimate the probability that the mean of a random sample of 40 observations of Y is more than 120. *(4 marks)*

(c) Indicate, with a reason, where, if at all, in this question you made use of the Central Limit Theorem. *(2 marks)*

January 2010

5 In a random sample of 12 bags of flour, the weight, in grams, of flour in each bag was recorded as follows.

1011 995 1018 1022 1014 1005 1017 1015 993 1018 992 1020

(a) It may be assumed that the weight of flour in a bag is normally distributed with a standard deviation of 10.5 grams.

(i) Construct a 98% confidence interval for the mean weight, μ grams, of flour in a bag, giving the limits to four significant figures. *(5 marks)*

(ii) State why, in constructing your confidence interval, use of the Central Limit Theorem was **not** necessary. *(1 mark)*

(iii) If the distribution of the weight of flour in a bag was unknown, indicate a minimum number of weights that you would consider necessary for a confidence interval for μ to be valid. *(1 mark)*

(b) The statement '1 kg' is printed on each bag.

Comment on this statement using **both** the confidence interval that you constructed in part (a)(i) and the weights of the given sample of 12 bags. *(3 marks)*

(c) Given that $\mu = 1000$, state the probability that a 98% confidence interval for μ will **not** contain 1000. *(1 mark)*

June 2010

7 An ambulance control centre responds to emergency calls in a rural area. The response time, T minutes, is defined as the time between the answering of an emergency call at the centre and the arrival of an ambulance at the given location of the emergency.

Response times have an unknown mean μ_T and an unknown variance.

Anita, the centre's manager, asked Peng, a student on supervised work experience, to record and summarise the values of T obtained from a random sample of 80 emergency calls.

Peng's summarised results were

Mean, $\bar{t} = 6.31$ Variance (unbiased estimate), $s^2 = 19.3$

Only 1 of the 80 values of T exceeded 20

(a) Anita then asked Peng to determine a confidence interval for μ_T. Peng replied that, from his summarised results, T was **not** normally distributed and so a valid confidence interval for μ_T could **not** be constructed.

(i) Explain, using the value of $\bar{t} - 2s$, why Peng's conclusion that T was not normally distributed was likely to be **correct**. *(2 marks)*

(ii) Explain why Peng's conclusion that a valid confidence interval for μ_T could not be constructed was **incorrect**. *(2 marks)*

(b) Construct a 98% confidence interval for μ_T. *(4 marks)*

(c) Anita had two targets for T. These were that $\mu_T < 8$ and that $P(T \leqslant 20) > 95\%$.

Indicate, with justification, whether **each** of these two targets was likely to have been met. *(3 marks)*

3 The volume, X litres, of orange juice in a 1-litre carton may be modelled by a normal distribution with unknown mean μ.

The volumes, x litres, recorded to the nearest 0.01 litre, in a random sample of 100 cartons are shown in the table.

Volume (x litres)	**Number of cartons (f)**
0.95 – 0.97	2
0.98 – 1.00	7
1.01 – 1.03	15
1.04 – 1.06	32
1.07 – 1.09	22
1.10 – 1.12	14
1.13 – 1.15	7
1.16 – 1.18	1
Total	**100**

(a) For the group '0.98 – 1.00':

(i) show that it has a mid-point of 0.99 litres;

(ii) state the minimum and the maximum values of x that could be included in this group. *(2 marks)*

(b) Calculate, to three decimal places, estimates of the mean and the standard deviation of these 100 volumes. *(3 marks)*

(c) (i) Construct an approximate 99% confidence interval for μ. *(4 marks)*

(ii) State why use of the Central Limit Theorem was **not** required when calculating this confidence interval. *(1 mark)*

(iii) Give a reason why the confidence interval is approximate rather than exact. *(1 mark)*

(d) Give a reason in support of the claim that:

(i) $\mu > 1$;

(ii) $P(0.94 < X < 1.16)$ is approximately 1. *(2 marks)*

4 Rice that can be cooked in microwave ovens is sold in packets which the manufacturer claims contain a mean weight of more than 250 grams of rice.

The weight of rice in a packet may be modelled by a normal distribution.

A consumer organisation's researcher weighed the contents, x grams, of each of a random sample of 50 packets. Her summarised results are:

$$\bar{x} = 251.1 \quad \text{and} \quad \sum(x - \bar{x})^2 = 184.5$$

(a) Show that, correct to two decimal places, $s = 1.94$, where s^2 denotes the unbiased estimate of the population variance. *(1 mark)*

(b) (i) Construct a 96% confidence interval for the mean weight of rice in a packet, giving the limits to one decimal place. *(4 marks)*

(ii) Hence comment on the manufacturer's claim. *(2 marks)*

(c) The statement '250 grams' is printed on each packet.

Explain, with reference to the values of $\bar{x}$ and s, why the consumer organisation may consider this statement to be dubious. *(2 marks)*

January 2012

7 A random sample of 50 full-time university employees was selected as part of a higher education salary survey.

The annual salary in thousands of pounds, x, of each employee was recorded, with the following summarised results.

$$\sum x = 2290.0 \quad \text{and} \quad \sum (x - \bar{x})^2 = 28\,225.50$$

Also recorded was the fact that 6 of the 50 salaries exceeded £60 000.

(a) (i) Calculate values for $\bar{x}$ and s, where s^2 denotes the unbiased estimate of σ^2. *(3 marks)*

(ii) Hence show why the annual salary, X, of a full-time university employee is unlikely to be normally distributed. Give numerical support for your answer. *(2 marks)*

(b) (i) Indicate why the mean annual salary, $\overline{X}$, of a random sample of 50 full-time university employees may be assumed to be normally distributed. *(2 marks)*

(ii) Hence construct a 99% confidence interval for the mean annual salary of full-time university employees. *(4 marks)*

(c) It is claimed that the annual salaries of full-time university employees have an average which exceeds £55 000 and that more than 25% of such salaries exceed £60 000.

Comment on **each** of these two claims. *(3 marks)*

June 2012

7 The volume of bleach in a 5-litre bottle may be modelled by a random variable with a standard deviation of 75 millilitres.

The volume, in litres, of bleach in each of a random sample of 36 such bottles was measured. The 36 measurements resulted in a **total** volume of 181.80 litres and exactly 8 bottles contained less than 5 litres.

(a) Construct a 98% confidence interval for the mean volume of bleach in a 5-litre bottle. *(5 marks)*

(b) It is claimed that the mean volume of bleach in a 5-litre bottle exceeds 5 litres and also that fewer than 10 per cent of such bottles contain less than 5 litres.

Comment, with numerical justification, on **each** of these two claims. *(3 marks)*

(c) State, with justification, whether you made use of the Central Limit Theorem in constructing the confidence interval in part **(a)**. *(1 mark)*

January 2013

6 (a) The length of one-metre galvanised-steel straps used in house building may be modelled by a normal distribution with a mean of 1005 mm and a standard deviation of 15 mm.

The straps are supplied to house builders in packs of 12, and the straps in a pack may be assumed to be a random sample.

Determine the probability that the **mean** length of straps in a pack is less than one metre. *(4 marks)*

(b) Tania, a purchasing officer for a nationwide house builder, measures the **thickness**, x millimetres, of each of a random sample of 24 galvanised-steel straps supplied by a manufacturer. She then calculates correctly that the value of $\bar{x}$ is 4.65 mm.

(i) Assuming that the thickness, X mm, of such a strap may be modelled by the distribution $N(\mu, 0.15^2)$, construct a 99% confidence interval for μ. *(4 marks)*

(ii) Hence comment on the manufacturer's specification that the mean thickness of such straps is greater than 4.5 mm. *(2 marks)*

June 2013

6 The weight, X kilograms, of sand in a bag can be modelled by a normal random variable with unknown mean μ and known standard deviation 0.4.

(a) The sand in each of a random sample of 25 bags from a large batch is weighed. The **total** weight of sand in these 25 bags is found to be 497.5 kg.

(i) Construct a 98% confidence interval for the mean weight of sand in bags in the batch. *(5 marks)*

(ii) Hence comment on the claim that bags in the batch contain an average of 20 kg of sand. *(2 marks)*

(iii) State why use of the Central Limit Theorem is **not** required in answering part **(a)(i)**. *(1 mark)*

(b) The weight, Y kilograms, of cement in a bag can be modelled by a normal random variable with mean 25.25 and standard deviation 0.35.

A firm purchases 10 such bags. These bags may be considered to be a random sample.

(i) Determine the probability that the **mean** weight of cement in the 10 bags is **less** than 25 kg. *(4 marks)*

(ii) Calculate the probability that the weight of cement in **each** of the 10 bags is **more** than 25 kg. *(4 marks)*