S1: Binomial Distribution

Past Paper Questions
2006 - 2013

Name:

January 2006

6 Plastic clothes pegs are made in various colours.

The number of red pegs may be modelled by a binomial distribution with parameter p equal to 0.2.

The contents of packets of 50 pegs of mixed colours may be considered to be random samples.

(a) Determine the probability that a packet contains:

(i) less than or equal to 15 red pegs; *(2 marks)*

(ii) exactly 10 red pegs; *(2 marks)*

(iii) more than 5 but fewer than 15 red pegs. *(3 marks)*

(b) Sly, a student, claims to have counted the number of red pegs in each of 100 packets of 50 pegs. From his results the following values are calculated.

Mean number of red pegs per packet = 10.5

Variance of number of red pegs per packet = 20.41

Comment on the validity of Sly's claim. *(4 marks)*

June 2006

5 Kirk and Les regularly play each other at darts.

(a) The probability that Kirk wins any game is 0.3, and the outcome of each game is independent of the outcome of every other game.

Find the probability that, in a match of 15 games, Kirk wins:

(i) exactly 5 games; *(3 marks)*

(ii) fewer than half of the games; *(3 marks)*

(iii) more than 2 but fewer than 7 games. *(3 marks)*

(b) Kirk attends darts coaching sessions for three months. He then claims that he has a probability of 0.4 of winning any game, and that the outcome of each game is independent of the outcome of every other game.

(i) Assuming this claim to be true, calculate the mean and standard deviation for the number of games won by Kirk in a match of 15 games. *(3 marks)*

(ii) To assess Kirk's claim, Les keeps a record of the number of games won by Kirk in a series of 10 matches, each of 15 games, with the following results:

8 5 6 3 9 12 4 2 6 5

Calculate the mean and standard deviation of these values. *(2 marks)*

(iii) Hence comment on the validity of Kirk's claim. *(3 marks)*

January 2007

2 A hotel has 50 single rooms, 16 of which are on the ground floor. The hotel offers guests a choice of a full English breakfast, a continental breakfast or no breakfast. The probabilities of these choices being made are 0.45, 0.25 and 0.30 respectively. It may be assumed that the choice of breakfast is independent from guest to guest.

(a) On a particular morning there are 16 guests, each occupying a single room on the ground floor. Calculate the probability that exactly 5 of these guests require a full English breakfast. *(3 marks)*

(b) On a particular morning when there are 50 guests, each occupying a single room, determine the probability that:

(i) at most 12 of these guests require a continental breakfast; *(2 marks)*

(ii) more than 10 but fewer than 20 of these guests require no breakfast. *(3 marks)*

(c) When there are 40 guests, each occupying a single room, calculate the mean and the standard deviation for the number of these guests requiring breakfast. *(4 marks)*

June 2007

6 Each weekday, Monday to Friday, Trina catches a train from her local station. She claims that the probability that the train arrives on time at the station is 0.4, and that the train's arrival time is independent from day to day.

(a) Assuming her claims to be true, determine the probability that the train arrives on time at the station:

(i) on at most 3 days during a 2-week period (10 days); *(2 marks)*

(ii) on more than 10 days but fewer than 20 days during an 8-week period. *(3 marks)*

(b) (i) Assuming Trina's claims to be true, determine the mean and standard deviation for the number of times during a week (5 days) that the train arrives on time at the station. *(3 marks)*

(ii) Each week, for a period of 13 weeks, Trina's travelling colleague, Suzie, records the number of times that the train arrives on time at the station. Suzie's results are

2 2 4 1 2 3 3 2 2 0 3 2 0

Calculate the mean and standard deviation of these values. *(3 marks)*

(iii) Hence comment on the likely validity of Trina's claims. *(2 marks)*

January 2008

7 A travel agency in Tunisia offers customers a 3-day tour into the Sahara desert by either coach or minibus.

(a) The agency accepts bookings from 50 customers for seats on the coach. The probability that a customer, who has booked a seat on the coach, will **not** turn up to claim the seat is 0.08, and may be assumed to be independent of the behaviour of other customers.

Determine the probability that, of the customers who have booked a seat on the coach:

(i) two or more will **not** turn up;

(ii) three or more will **not** turn up. *(4 marks)*

(b) The agency accepts bookings from 15 customers for seats on the minibus. The probability that a customer, who has booked a seat on the minibus, will **not** turn up to claim the seat is 0.025, and may be assumed to be independent of the behaviour of other customers.

Calculate the probability that, of the customers who have booked a seat on the minibus:

(i) all will turn up;

(ii) one or more will **not** turn up. *(4 marks)*

(c) The coach has 48 seats and the minibus has 14 seats. If 14 or fewer customers who have booked seats on the minibus turn up, they will be allocated a seat on the minibus. If all 15 customers who have booked seats on the minibus turn up, one will be allocated a seat on the coach. This will leave only 47 seats available for the 50 customers who have booked seats on the coach.

Use your results from parts (a) and (b) to calculate the probability that there will be seats available on the coach for all those who turn up having booked such seats. *(4 marks)*

June 2008

6 For the adult population of the UK, 35 per cent of men and 29 per cent of women do not wear glasses or contact lenses.

(a) Determine the probability that, in a random sample of 40 men:

(i) at most 15 do not wear glasses or contact lenses; *(3 marks)*

(ii) more than 10 but fewer than 20 do not wear glasses or contact lenses. *(3 marks)*

(b) Calculate the probability that, in a random sample of 10 women, exactly 3 do not wear glasses or contact lenses. *(3 marks)*

(c) (i) Calculate the mean and the variance for the number who **do** wear glasses or contact lenses in a random sample of 20 women. *(3 marks)*

(ii) The numbers wearing glasses or contact lenses in 10 groups, each of 20 women, had a mean of 16.5 and a variance of 2.50.

Comment on the claim that these 10 groups were **not** random samples. *(3 marks)*

January 2009

7 The proportion of passengers who use senior citizen bus passes to travel into a particular town on 'Park & Ride' buses between 9.30 am and 11.30 am on weekdays is 0.45.

It is proposed that, when there are n passengers on a bus, a suitable model for the number of passengers using senior citizen bus passes is the distribution $B(n, 0.45)$.

(a) Assuming that this model applies to the 10.30 am weekday 'Park & Ride' bus into the town:

(i) calculate the probability that, when there are **16** passengers, exactly 3 of them are using senior citizen bus passes; *(3 marks)*

(ii) determine the probability that, when there are **25** passengers, fewer than 10 of them are using senior citizen bus passes; *(2 marks)*

(iii) determine the probability that, when there are **40** passengers, at least 15 but at most 20 of them are using senior citizen bus passes; *(3 marks)*

(iv) calculate the mean and the variance for the number of passengers using senior citizen bus passes when there are **50** passengers. *(2 marks)*

(b) (i) Give a reason why the proposed model may not be suitable. *(1 mark)*

(ii) Give a **different** reason why the proposed model would not be suitable for the number of passengers using senior citizen bus passes to travel into the town on the **7.15 am** weekday 'Park & Ride' bus. *(1 mark)*

June 2009

7 Mr Alott and Miss Fewer work in a postal sorting office.

(a) The number of letters per batch, R, sorted incorrectly by Mr Alott when sorting batches of 50 letters may be modelled by the distribution $B(50, 0.15)$.

Determine:

(i) $P(R < 10)$;

(ii) $P(5 \leqslant R \leqslant 10)$. *(4 marks)*

(b) It is assumed that the probability that Miss Fewer sorts a letter incorrectly is 0.06, and that her sorting of a letter incorrectly is independent from letter to letter.

(i) Calculate the probability that, when sorting a batch of **22** letters, Miss Fewer sorts exactly 2 letters incorrectly. *(3 marks)*

(ii) Calculate the probability that, when sorting a batch of **35** letters, Miss Fewer sorts at least 1 letter incorrectly. *(2 marks)*

(iii) Calculate the mean and the variance for the number of letters sorted **correctly** by Miss Fewer when she sorts a batch of **120** letters. *(2 marks)*

(iv) Miss Fewer sorts a random sample of 20 batches, each containing 120 letters. The number of letters sorted **correctly** per batch has a mean of 112.8 and a variance of 56.86.

Comment on the assumptions that the probability that Miss Fewer sorts a letter incorrectly is 0.06, and that her sorting of a letter incorrectly is independent from letter to letter. *(3 marks)*

January 2010

6 During the winter, the probability that Barry's cat, Sylvester, chooses to stay outside all night is 0.35, and the cat's choice is independent from night to night.

(a) Determine the probability that, during a period of 2 weeks (14 nights) in winter, Sylvester chooses to stay outside:

(i) on at most 7 nights; *(2 marks)*

(ii) on at least 11 nights; *(2 marks)*

(iii) on more than 5 nights but fewer than 10 nights. *(3 marks)*

(b) Calculate the probability that, during a period of **3 weeks** in winter, Sylvester chooses to stay outside on exactly 4 nights. *(3 marks)*

(c) Barry claims that, during the summer, the number of nights per week, S, on which Sylvester chooses to stay outside can be modelled by a binomial distribution with $n = 7$ and $p = \frac{5}{7}$.

(i) Assuming that Barry's claim is correct, find the mean and the variance of S. *(2 marks)*

(ii) For a period of 13 weeks during the summer, the number of nights per week on which Sylvester chose to stay outside had a mean of 5 and a variance of 1.5.

Comment on Barry's claim. *(2 marks)*

June 2010

4 In a certain country, 15 per cent of the male population is left-handed.

(a) Determine the probability that, in a random sample of 50 males from this country:

(i) at most 10 are left-handed; *(2 marks)*

(ii) at least 5 are left-handed; *(2 marks)*

(iii) more than 6 but fewer than 12 are left-handed. *(3 marks)*

(b) In the same country, 11 per cent of the female population is left-handed.

Calculate the probability that, in a random sample of 35 females from this country, exactly 4 are left-handed. *(3 marks)*

(c) A sample of 2000 people is selected at random from the population of the country. The proportion of males in the sample is 52 per cent.

How many people in the sample would you expect to be left-handed? *(4 marks)*

4 Clay pigeon shooting is the sport of shooting at special flying clay targets with a shotgun.

(a) Rhys, a novice, uses a single-barrelled shotgun. The probability that he hits a target is 0.45, and may be assumed to be independent from target to target.

Determine the probability that, in a series of shots at 15 targets, he hits:

(i) at most 5 targets; *(1 mark)*

(ii) more than 10 targets; *(2 marks)*

(iii) exactly 6 targets; *(2 marks)*

(iv) at least 5 but at most 10 targets. *(3 marks)*

(b) Sasha, an expert, uses a double-barrelled shotgun. She shoots at each target with the gun's first barrel and, only if she misses, does she then shoot at the target with the gun's second barrel.

The probability that she hits a target with a shot using her gun's first barrel is 0.85. The conditional probability that she hits a target with a shot using her gun's second barrel, given that she has missed the target with a shot using her gun's first barrel, is 0.80. Assume that Sasha's shooting is independent from target to target.

(i) Show that the probability that Sasha hits a target is 0.97. *(2 marks)*

(ii) Determine the probability that, in a series of shots at 50 targets, Sasha hits at least 48 targets. *(3 marks)*

(iii) In a series of shots at 80 targets, calculate the mean number of times that Sasha shoots at targets with her gun's second barrel. *(2 marks)*

June 2011

6 An amateur tennis club purchases tennis balls that have been used previously in professional tournaments.

The probability that each such ball fails a standard bounce test is 0.15.

The club purchases boxes each containing 10 of these tennis balls. Assume that the 10 balls in any box represent a random sample.

(a) Determine the probability that the number of balls in a box which fail the bounce test is:

(i) at most 2; *(1 mark)*

(ii) at least 2; *(2 marks)*

(iii) more than 1 but fewer than 5. *(3 marks)*

(b) Determine the probability that, in **5 boxes**, the total number of balls which fail the bounce test is:

(i) more than 5; *(2 marks)*

(ii) at least 5 but at most 10. *(3 marks)*

January 2012

4 The records at a passport office show that, on average, 15 per cent of photographs that accompany applications for passport renewals are unusable.

Assume that exactly one photograph accompanies each application.

(a) Determine the probability that in a random sample of 40 applications:

(i) exactly 6 photographs are unusable;

(ii) at most 5 photographs are unusable;

(iii) more than 5 but fewer than 10 photographs are unusable. *(7 marks)*

(b) Calculate the mean and the standard deviation for the number of photographs that are unusable in a random sample of **32** applications. *(3 marks)*

(c) Mr Stickler processes 32 applications each day. His records for the previous 10 days show that the numbers of photographs that he deemed unusable were

8 6 10 7 9 7 8 9 6 7

By calculating the mean and the standard deviation of these values, comment, with reasons, on the suitability of the $B(32, 0.15)$ model for the number of photographs deemed unusable each day by Mr Stickler. *(4 marks)*

6 A bin contains a very large number of paper clips of different colours. The proportion of each colour is shown in the table.

Colour	**White**	**Yellow**	**Green**	**Blue**	**Red**	**Purple**
Proportion	0.15	0.15	0.20	0.15	0.22	0.13

(a) Packets are filled from the bin. Each filled packet contains exactly 30 paper clips which may be considered to be a random sample.

Use binomial distributions to determine the probability that a filled packet contains:

(i) exactly 2 purple paper clips; *(3 marks)*

(ii) a **total** of more than 10 red or purple paper clips; *(3 marks)*

(iii) at least 5 but at most 10 green paper clips. *(3 marks)*

(b) Jumbo packets are also filled from the bin. Each filled jumbo packet contains exactly 100 paper clips.

(i) Assuming that the number of paper clips in a jumbo packet may be considered to be a random sample, calculate the mean and the variance of the number of **red** paper clips in a filled jumbo packet. *(2 marks)*

(ii) It is claimed that the proportion of red paper clips in the bin is greater than 0.22 and that jumbo packets do not contain random samples of paper clips.

An analysis of the number of red paper clips in each of a random sample of 50 filled jumbo packets resulted in a mean of 22.1 and a standard deviation of 4.17.

Comment, with numerical justification, on **each** of the two claims. *(3 marks)*

3 *Stopoff* owns a chain of hotels. Guests are presented with the bills for their stays when they check out.

(a) Assume that the number of bills that contain errors may be modelled by a binomial distribution with parameters n and p, where $p = 0.30$.

Determine the probability that, in a random sample of 40 bills:

(i) at most 10 bills contain errors;

(ii) at least 15 bills contain errors;

(iii) exactly 12 bills contain errors. *(6 marks)*

(b) Calculate the mean and the variance for **each** of the distributions B(16, 0.20) and B(16, 0.125). *(3 marks)*

(c) Stan, who is a travelling salesperson, always uses *Stopoff* hotels. He holds one of its diamond customer cards and so should qualify for special customer care. However, he regularly finds errors in his bills when he checks out.

Each month, during a 12-month period, Stan stayed in *Stopoff* hotels on exactly 16 occasions. He recorded, each month, the number of occasions on which his bill contained errors. His recorded values were as follows.

2 1 4 3 1 3 0 3 1 0 5 1

(i) Calculate the mean and the variance of these 12 values. *(2 marks)*

(ii) Hence state with reasons which, if either, of the distributions B(16, 0.20) and B(16, 0.125) is likely to provide a satisfactory model for these 12 values. *(3 marks)*

3 An auction house offers items of jewellery for sale at its public auctions. Each item has a reserve price which is less than the lower price estimate which, in turn, is less than the upper price estimate. The outcome for any item is independent of the outcomes for all other items.

The auction house has found, from past records, the following probabilities for the outcomes of items of jewellery offered for sale.

Outcome	Probability
Item does not achieve its reserve price	0.15
Item achieves at least its reserve price	0.85
Item achieves at least its lower price estimate	0.50
Item achieves at least its upper price estimate	0.175

For example, the probability that an item achieves at least its lower price estimate but not its upper price estimate is 0.325 .

A particular auction includes exactly 40 items of jewellery that may be assumed to be a random sample of such items.

(a) Use binomial distributions to find the probability that:

(i) at most 10 items do not achieve their reserve prices; *(1 mark)*

(ii) 25 or more items achieve at least their lower price estimates; *(2 marks)*

(iii) exactly 2 items achieve at least their upper price estimates; *(2 marks)*

(iv) more than 10 items but fewer than 15 items achieve at least their reserve prices but not their lower price estimates. *(4 marks)*

(b) How many of the 40 items of jewellery would you expect to achieve at least their reserve prices but not their upper price estimates? *(2 marks)*