Core 1: Polynomials

Past Exam Questions
2006 - 2013

Name:

January 2006

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Hence express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

June 2006

6 The polynomial $p(x)$ is given by $p(x) = x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when $p(x)$ is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express $p(x)$ in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

January 2007

1 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $p(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

June 2007

6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x - 1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

January 2008

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

June 2008

6 The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$.

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of linear factors. *(3 marks)*

(c) (i) The curve with equation $y = x^3 + x^2 - 8x - 12$ passes through the point $(0, k)$. State the value of k. *(1 mark)*

(ii) Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis. *(3 marks)*

January 2009

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

June 2009

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

January 2010

1 The polynomial $p(x)$ is given by $p(x) = x^3 - 13x - 12$.

(a) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(b) Express $p(x)$ as the product of three linear factors. *(3 marks)*

June 2010

3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 2$. *(2 marks)*

(c) (i) Verify that $p(-1) < p(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

January 2011

5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x+1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x-3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Express $\mathrm{p}(x)$ in the form $(x-3)(x^2+bx+c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

June 2011

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 2x^2 + 3$.

(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x-3$. *(2 marks)*

(b) Use the Factor Theorem to show that $x+1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(c) (i) Express $\mathrm{p}(x) = x^3 - 2x^2 + 3$ in the form $(x+1)(x^2+bx+c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. *(2 marks)*

January 2012

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $\mathrm{p}(x)$ is divided by $x+2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x-3$ is a factor of $\mathrm{p}(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

June 2012

3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 2x^2 - 5x - 6$$

(a) (i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $p(0) > p(1)$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 + 2x^2 - 5x - 6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

January 2013

5 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

June 2013

4 (a) The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$ *(1 mark)*

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$. *(2 marks)*

(iv) Find the value of $\frac{d^2y}{dx^2}$ when $x = -3$. *(3 marks)*

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$. *(1 mark)*