M3: Projectile on an incline

Past Paper Questions
2006 - 2013

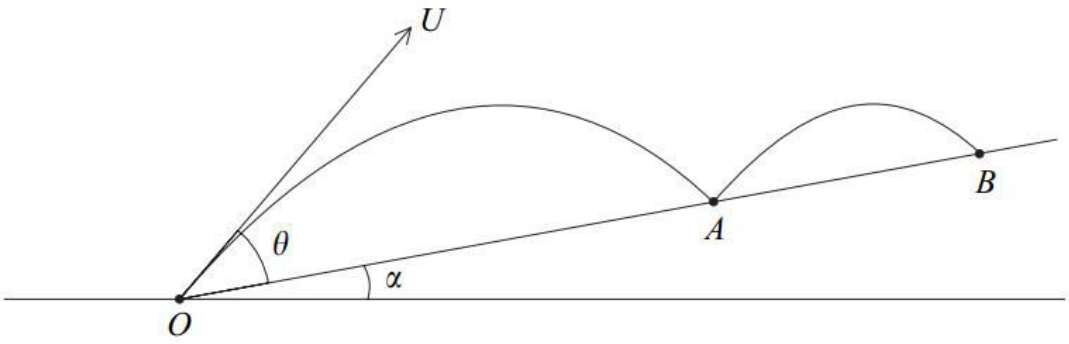
Name:

7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

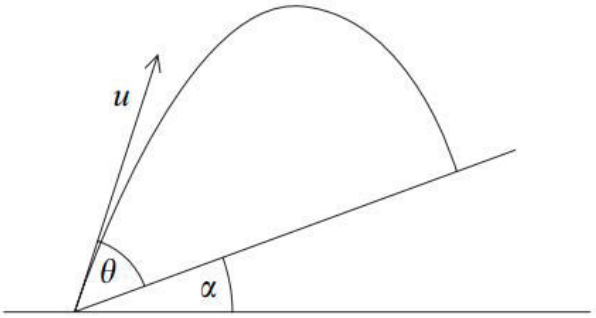
(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*

(b) The projectile rebounds and strikes the hill again at a point B. The hill is smooth and the coefficient of restitution between the projectile and the hill is e.



Find the ratio of the time of flight from O to A to the time of flight from A to B. Give your answer in its simplest form. *(4 marks)*

7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

(8 marks)

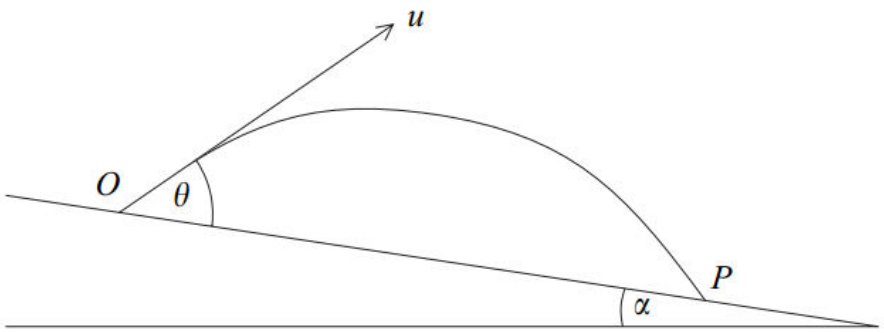
(b) Hence, using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2 \tan\theta = \cot\alpha$$

(4 marks)

7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, θ and α, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

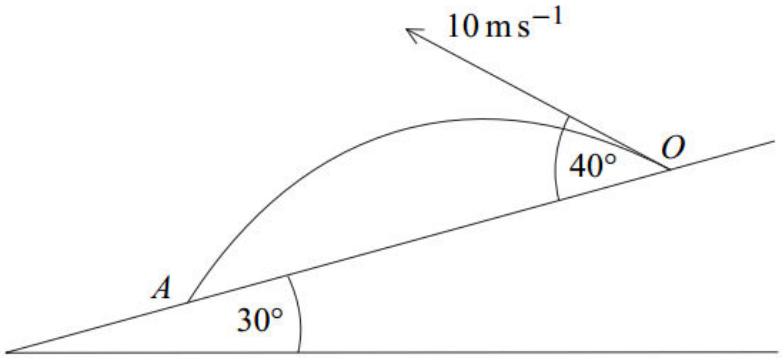
(b) (i) Find, in terms of u, g, θ and α, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin\theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$. *(6 marks)*

(iii) Hence, by using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin\alpha)}$. *(5 marks)*

June 2009

7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity $10\,\text{m s}^{-1}$ at an angle of 40° above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

$$\frac{20 \sin 40°}{g \cos 30°}$$

(3 marks)

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5. Find the speed of the particle as it rebounds from the slope. *(4 marks)*

June 2010

7 A ball is projected from a point O on a smooth plane which is inclined at an angle of 35° above the horizontal. The ball is projected with velocity $20\,\text{m s}^{-1}$ at an angle of 30° above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.

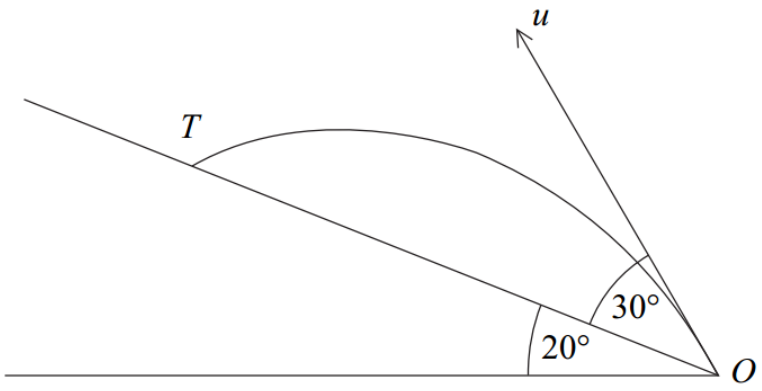
(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

6 A projectile is fired from a point O on a plane which is inclined at an angle of 20° to the horizontal. The projectile is fired up the plane with velocity $u\,\text{m s}^{-1}$ at an angle of 30° to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

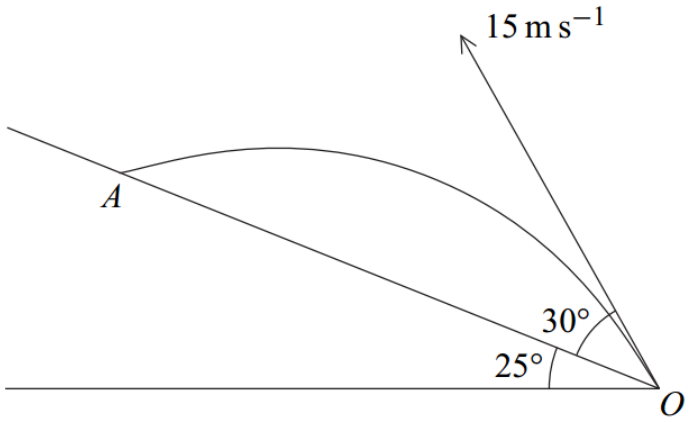
The projectile hits a target T on the inclined plane.



(a) Given that $OT = 200\,\text{m}$, determine the value of u. *(7 marks)*

(b) Find the greatest perpendicular distance of the projectile from the inclined plane. *(4 marks)*

5 A particle is projected from a point O on a smooth plane, which is inclined at 25° to the horizontal. The particle is projected up the plane with velocity $15\,\text{m s}^{-1}$ at an angle 30° above the plane. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.

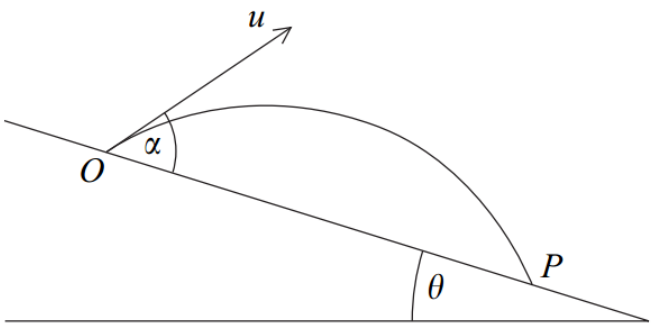


(a) Find the time taken by the particle to travel from O to A. *(4 marks)*

(b) The coefficient of restitution between the particle and the inclined plane is $\frac{2}{3}$.

Find the speed of the particle as it rebounds from the inclined plane at A. *(8 marks)*

5 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected down the plane with velocity u at an angle α above the plane. The particle first strikes the plane at a point P, as shown in the diagram. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Given that the time of flight from O to P is T, find an expression for u in terms of θ, α, T and g. *(4 marks)*

(b) Using the identity $\cos(X - Y) = \cos X \cos Y + \sin X \sin Y$, show that the distance OP is given by $\dfrac{2u^2 \sin\alpha \cos(\alpha - \theta)}{g\cos^2\theta}$. *(6 marks)*