M3: Path of a Projectile

Past Paper Questions
2006 - 2013

Name:

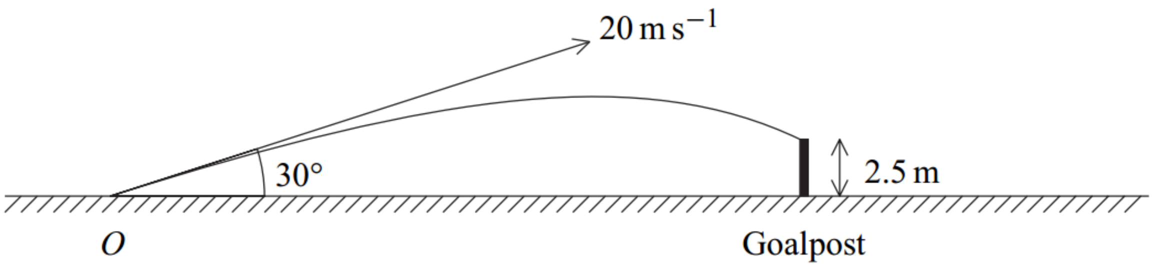
June 2006

5 A football is kicked from a point O on a horizontal football ground with a velocity of $20\,\text{m}\,\text{s}^{-1}$ at an angle of elevation of $30°$. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = x \tan 30° - \frac{gx^2}{800 \cos^2 30°}$$ *(6 marks)*

(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal. *(4 marks)*



June 2007

5 A ball is projected with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 \alpha)$$ *(6 marks)*

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\text{m}\,\text{s}^{-2}$, show that R satisfies the equation

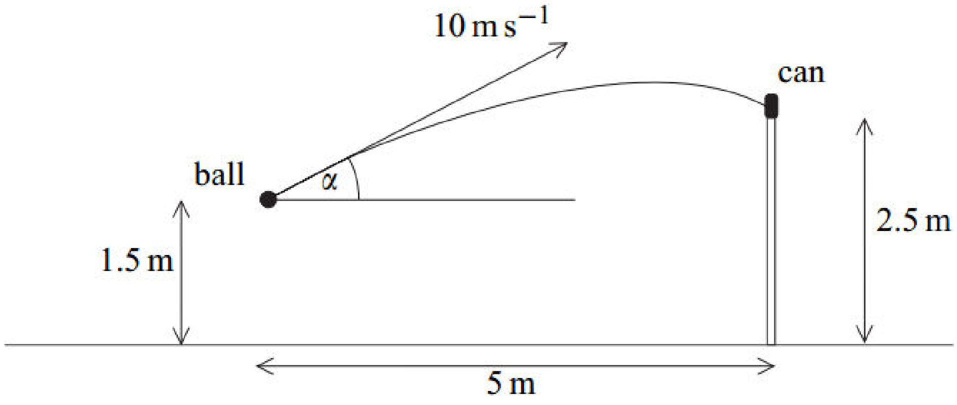
$$5R^2 \tan^2 \alpha - u^2 R \tan \alpha + 5R^2 + u^2 = 0$$ *(2 marks)*

(ii) Hence, given that u and R are constants, show that, for $\tan \alpha$ to have real values, R must satisfy the inequality

$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$ *(2 marks)*

June 2008

5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity 10 m s^{-1} at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49 \tan^2 \alpha - 200 \tan \alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than 8 m s^{-1}.

Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

June 2009

2 A particle is projected from a point O on a horizontal plane and has initial velocity components of 2 m s^{-1} and 10 m s^{-1} parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

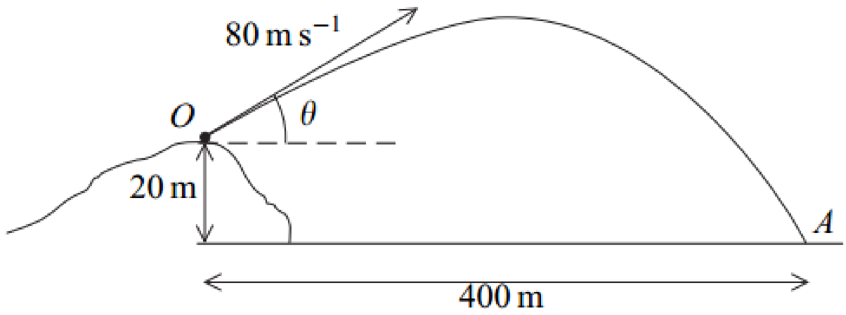
(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}(1 + \tan^2\theta)$$ *(5 marks)*

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

$$49\tan^2\theta - 160\tan\theta + 41 = 0$$ *(2 marks)*

(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

June 2011

3 **(In this question, use $g = 10\,\text{m}\,\text{s}^{-2}$.)**

A golf ball is hit from a point O on a horizontal golf course with a velocity of $40\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^2 \tan^2 \theta - 320x \tan \theta + (x^2 + 320y) = 0$$ *(6 marks)*

(b) (i) The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of θ. *(5 marks)*

(ii) Which value of θ gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of θ. *(2 marks)*

June 2012

3 **(In this question, take $g = 10\,\text{m}\,\text{s}^{-2}$.)**

A projectile is fired from a point O with speed u at an angle of elevation α above the horizontal so as to pass through a point P. The projectile travels in a vertical plane through O and P. The point P is at a horizontal distance $2k$ from O and at a vertical distance k above O.

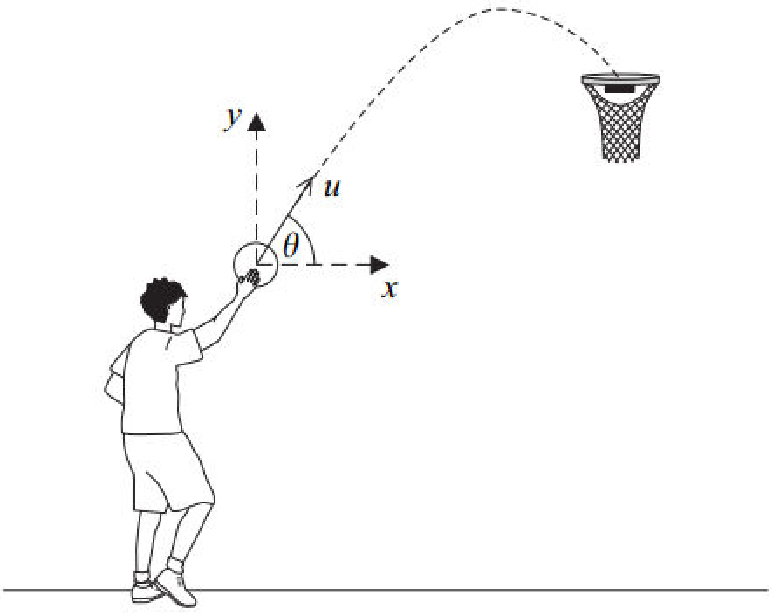
(a) Show that α satisfies the equation

$$20k \tan^2 \alpha - 2u^2 \tan \alpha + u^2 + 20k = 0$$ *(7 marks)*

(b) Deduce that

$$u^4 - 20ku^2 - 400k^2 \geqslant 0$$ *(3 marks)*

3 A player projects a basketball with speed $u \text{ m s}^{-1}$ at an angle θ above the horizontal. The basketball travels in a vertical plane through the point of projection and goes into the basket. During the motion, the horizontal and upward vertical displacements of the basketball from the point of projection are x metres and y metres respectively.



(a) Find an expression for y in terms of x, u, g and $\tan\theta$. *(6 marks)*

(b) The player projects the basketball with speed 8 m s^{-1} from a point 0.5 metres vertically below and 5 metres horizontally from the basket.

(i) Show that the two possible values of θ are approximately 63.1° and 32.6°, correct to three significant figures. *(5 marks)*

(ii) Given that the player projects the basketball at 63.1° to the horizontal, find the direction of the motion of the basketball as it enters the basket. Give your answer to the nearest degree. *(4 marks)*

(c) State a modelling assumption needed for answering parts **(a)** and **(b)** of this question. *(1 mark)*