M2: Differential Equations

Past Paper Questions
2006 - 2013

Name:

January 2006

6 A student is modelling the motion of a small boat as it moves on a lake. When the speed of the boat is $12\,\text{m s}^{-1}$, the engine is switched off. At time t seconds later, it has a velocity of $v\,\text{m s}^{-1}$ and experiences a resistance force of magnitude $20v$ newtons. The mass of the boat is 80 kg.

To set up a simple model for the motion of the boat, the student assumes that the water in the lake is still and that the boat travels in a straight line.

(a) Explain how these two assumptions allow the student to create a simple model. *(2 marks)*

(b) State one other assumption that the student should make. *(1 mark)*

(c) (i) Express $\frac{dv}{dt}$ in terms of v. *(2 marks)*

(ii) Find an expression for v in terms of t. *(5 marks)*

June 2006

7 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v\,\text{m s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\,\text{m s}^{-1}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4}\right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

January 2007

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\frac{dv}{dt} = -\frac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(6 marks)*

June 2007

7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{dv}{dt} = -\lambda v$$ *(2 marks)*

(b) The initial speed of the stone is U.

Show that

$$v = Ue^{-\lambda t}$$ *(4 marks)*

January 2008

8 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v\,\text{m s}^{-1}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{dv}{dt} - \frac{8000}{v} + kv^2 = 0$$ *(4 marks)*

(b) When the velocity of the car is $20\,\text{m s}^{-1}$, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{dv}{dt} = -kv^2$$ *(1 mark)*

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to $10\,\text{m s}^{-1}$. *(5 marks)*

June 2008

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

$$\frac{dv}{dt} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\text{m s}^{-1}$.

Show that $v = 20e^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\text{m s}^{-1}$. *(3 marks)*

January 2009

8 A stone, of mass 0.05 kg, is moving along the smooth horizontal floor of a tank, which is filled with oil. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude $0.08v^2$.

(a) Show that

$$\frac{dv}{dt} = -1.6v^2$$

(2 marks)

(b) The initial speed of the stone is $3\,\text{m s}^{-1}$.

Show that

$$v = \frac{15}{5 + 24t}$$

(5 marks)

June 2009

8 A stone, of mass m, is moving in a straight line along smooth horizontal ground.

At time t, the stone has speed v. As the stone moves, it experiences a total resistance force of magnitude $\lambda m v^{\frac{3}{2}}$, where λ is a constant. No other horizontal force acts on the stone.

(a) Show that

$$\frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$$

(2 marks)

(b) The initial speed of the stone is $9\,\text{m s}^{-1}$.

Show that

$$v = \frac{36}{(2 + 3\lambda t)^2}$$

(7 marks)

(c) Find, in terms of λ, the time taken for the speed of the stone to drop to $4\,\text{m s}^{-1}$.

(3 marks)

January 2010

5 A golf ball, of mass m kg, is moving in a straight line across smooth horizontal ground. At time t seconds, the golf ball has speed $v\,\mathrm{m\,s^{-1}}$. As the golf ball moves, it experiences a resistance force of magnitude $0.2mv^{\frac{1}{2}}$ newtons until it comes to rest. No other horizontal force acts on the golf ball.

Model the golf ball as a particle.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.2v^{\frac{1}{2}}$$ *(1 mark)*

(b) When $t = 0$, the speed of the golf ball is $16\,\mathrm{m\,s^{-1}}$.

Show that $v = (4 - 0.1t)^2$. *(5 marks)*

(c) Find the value of t when $v = 1$. *(3 marks)*

(d) Find the distance travelled by the golf ball as its speed decreases from $16\,\mathrm{m\,s^{-1}}$ to $1\,\mathrm{m\,s^{-1}}$. *(4 marks)*

January 2011

8 Vicky has mass 65 kg and is skydiving. She steps out of a helicopter and falls vertically. She then waits a short period of time before opening her parachute. The parachute opens at time $t = 0$ when her speed is $19.6\,\mathrm{m\,s^{-1}}$, and she then experiences an air resistance force of magnitude $260v$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is her speed at time t seconds.

(a) When $t > 0$:

(i) show that the resultant downward force acting on Vicky is

$$65(9.8 - 4v) \text{ newtons}$$ *(1 mark)*

(ii) show that $\frac{\mathrm{d}v}{\mathrm{d}t} = -4(v - 2.45)$. *(2 marks)*

(b) By showing that $\int \frac{1}{v - 2.45}\,\mathrm{d}v = -\int 4\,\mathrm{d}t$, find v in terms of t. *(5 marks)*

June 2011

6 A car, of mass m kg, is moving along a straight horizontal road. At time t seconds, the car has speed $v\,\mathrm{m\,s^{-1}}$. As the car moves, it experiences a resistance force of magnitude $2mv^{\frac{5}{4}}$ newtons. No other horizontal force acts on the car.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -2v^{\frac{5}{4}}$$

(1 mark)

(b) The initial speed of the car is $16\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \left(\frac{2}{t+1}\right)^4$$

(5 marks)

January 2012

6 Alice places a toy, of mass 0.4 kg, on a slope. The toy is set in motion with an initial velocity of $1\,\mathrm{m\,s^{-1}}$ down the slope. The resultant force acting on the toy is $(2 - 4v)$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is the toy's velocity at time t seconds after it is set in motion.

(a) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = -10\,(v - 0.5)$. *(2 marks)*

(b) By using $\int \frac{1}{v - 0.5}\,\mathrm{d}v = -\int 10\,\mathrm{d}t$, find v in terms of t. *(5 marks)*

(c) Find the time taken for the toy's velocity to reduce to $0.55\,\mathrm{m\,s^{-1}}$. *(3 marks)*

June 2012

7 A stone, of mass 5 kg, is projected vertically downwards, in a viscous liquid, with an initial speed of $7\,\mathrm{m\,s^{-1}}$.

At time t seconds after it is projected, the stone has speed $v\,\mathrm{m\,s^{-1}}$ and it experiences a resistance force of magnitude $9.8v$ newtons.

(a) When $t \geqslant 0$, show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -1.96(v - 5)$$

(2 marks)

(b) Find v in terms of t. *(5 marks)*

6 A car accelerates from rest along a straight horizontal road.

The car's engine produces a constant horizontal force of magnitude 4000 N.

At time t seconds, the speed of the car is $v\,\text{m s}^{-1}$, and a resistance force of magnitude $40v$ newtons acts upon the car.

The mass of the car is 1600 kg.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{100 - v}{40}$. *(2 marks)*

(b) Find the velocity of the car at time t. *(6 marks)*