Core 4: Integration

Past Paper Questions
2006 - 2013

Name:

January 2006

6 (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where a and b are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where a is an integer. *(5 marks)*

June 2006

3 (a) Given that $\frac{9x^2 - 6x + 5}{(3x-1)(x-1)}$ can be written in the form $3 + \frac{A}{3x-1} + \frac{B}{x-1}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence, or otherwise, find $\int \frac{9x^2 - 6x + 5}{(3x-1)(x-1)} \, dx$. *(4 marks)*

January 2007

3 (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*

(b) (i) Hence show that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of x. *(2 marks)*

(ii) Solve the equation $3\sin x - \cos 2x = 1$ for $0° < x < 360°$. *(4 marks)*

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*

4 (a) (i) Express $\frac{3x-5}{x-3}$ in the form $A + \frac{B}{x-3}$, where A and B are integers. *(2 marks)*

(ii) Hence find $\int \frac{3x-5}{x-3} \, dx$. *(2 marks)*

(b) (i) Express $\frac{6x-5}{4x^2-25}$ in the form $\frac{P}{2x+5} + \frac{Q}{2x-5}$, where P and Q are integers. *(3 marks)*

(ii) Hence find $\int \frac{6x-5}{4x^2-25} \, dx$. *(3 marks)*

January 2008

1 (a) Given that $\frac{3}{9-x^2}$ can be expressed in the form $k\left(\frac{1}{3+x} + \frac{1}{3-x}\right)$, find the value of the rational number k. *(2 marks)*

(b) Show that $\int_1^2 \frac{3}{9-x^2} \, dx = \frac{1}{2}\ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

June 2008

3 (a)	By writing $\sin 3x$ as $\sin(x + 2x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$ for all values of x.	*(5 marks)*
(b)	Hence, or otherwise, find $\int \sin^3 x \, dx$.	*(3 marks)*

6 (a)	Express $\dfrac{2}{x^2 - 1}$ in the form $\dfrac{A}{x-1} + \dfrac{B}{x+1}$.	*(3 marks)*
(b)	Hence find $\displaystyle\int \frac{2}{x^2 - 1}\, dx$.	*(2 marks)*
(c)	Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2y}{3(x^2 - 1)}$, given that $y = 1$ when $x = 3$. Show that the solution can be written as $y^3 = \dfrac{2(x-1)}{x+1}$.	*(5 marks)*

January 2009

3 (a) (i)	Express $\dfrac{2x + 7}{x + 2}$ in the form $A + \dfrac{B}{x+2}$, where A and B are integers.	*(2 marks)*
(ii)	Hence find $\displaystyle\int \frac{2x+7}{x+2}\, dx$.	*(2 marks)*
(b) (i)	Express $\dfrac{28 + 4x^2}{(1 + 3x)(5 - x)^2}$ in the form $\dfrac{P}{1+3x} + \dfrac{Q}{5-x} + \dfrac{R}{(5-x)^2}$, where P, Q and R are constants.	*(5 marks)*
(ii)	Hence find $\displaystyle\int \frac{28 + 4x^2}{(1+3x)(5-x)^2}\, dx$.	*(4 marks)*

January 2010

4	The expression $\dfrac{10x^2 + 8}{(x+1)(5x-1)}$ can be written in the form $2 + \dfrac{A}{x+1} + \dfrac{B}{5x-1}$, where A and B are constants.	
(a)	Find the values of A and B.	*(4 marks)*
(b)	Hence find $\displaystyle\int \frac{10x^2 + 8}{(x+1)(5x-1)}\, dx$.	*(4 marks)*

June 2010

3 (a) (i) Express $\dfrac{7x-3}{(x+1)(3x-2)}$ in the form $\dfrac{A}{x+1}+\dfrac{B}{3x-2}$. *(3 marks)*

(ii) Hence find $\displaystyle\int \frac{7x-3}{(x+1)(3x-2)}\,dx$. *(2 marks)*

(b) Express $\dfrac{6x^2+x+2}{2x^2-x+1}$ in the form $P+\dfrac{Qx+R}{2x^2-x+1}$. *(3 marks)*

June 2011

4 (a) A curve is defined by the parametric equations $x=3\cos 2\theta$, $y=2\cos\theta$.

(i) Show that $\dfrac{dy}{dx}=\dfrac{1}{k\cos\theta}$, where k is an integer. *(4 marks)*

(ii) Find an equation of the normal to the curve at the point where $\theta=\dfrac{\pi}{3}$. *(4 marks)*

(b) Find the exact value of $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$. *(5 marks)*

January 2012

1 (a) Express $\dfrac{2x+3}{4x^2-1}$ in the form $\dfrac{A}{2x-1}+\dfrac{B}{2x+1}$, where A and B are integers. *(3 marks)*

(b) Express $\dfrac{12x^3-7x-6}{4x^2-1}$ in the form $Cx+\dfrac{D(2x+3)}{4x^2-1}$, where C and D are integers. *(3 marks)*

(c) Evaluate $\displaystyle\int_1^2 \frac{12x^3-7x-6}{4x^2-1}\,dx$, giving your answer in the form $p+\ln q$, where p and q are rational numbers. *(5 marks)*

June 2012

1 (a) (i) Express $\dfrac{5x-6}{x(x-3)}$ in the form $\dfrac{A}{x}+\dfrac{B}{x-3}$. *(2 marks)*

(ii) Find $\displaystyle\int \frac{5x-6}{x(x-3)}\,\mathrm{d}x$. *(2 marks)*

(b) (i) Given that

$$4x^3+5x-2=(2x+1)(2x^2+px+q)+r$$

find the values of the constants p, q and r. *(4 marks)*

(ii) Find $\displaystyle\int \frac{4x^3+5x-2}{2x+1}\,\mathrm{d}x$. *(3 marks)*

January 2013

5 (a) Find $\displaystyle\int x\sqrt{x^2+3}\,\mathrm{d}x$. *(2 marks)*

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x\sqrt{x^2+3}}{\mathrm{e}^{2y}}$$

given that $y=0$ when $x=1$. Give your answer in the form $y=\mathrm{f}(x)$. *(7 marks)*

June 2013

1 (a) (i) Express $\dfrac{5-8x}{(2+x)(1-3x)}$ in the form $\dfrac{A}{2+x}+\dfrac{B}{1-3x}$, where A and B are integers. *(3 marks)*

(ii) Hence show that $\displaystyle\int_{-1}^{0} \frac{5-8x}{(2+x)(1-3x)}\,\mathrm{d}x = p\ln 2$, where p is rational. *(4 marks)*

(b) (i) Given that $\dfrac{9-18x-6x^2}{2-5x-3x^2}$ can be written as $C+\dfrac{5-8x}{2-5x-3x^2}$, find the value of C. *(1 mark)*

(ii) Hence find the exact value of the area of the region bounded by the curve $y=\dfrac{9-18x-6x^2}{2-5x-3x^2}$, the x-axis and the lines $x=-1$ and $x=0$.

You may assume that $y>0$ when $-1\leqslant x\leqslant 0$. *(2 marks)*