FP2: Roots of Polynomials

Past Paper Questions
2006 - 2013

Name:

January 2006

2 The cubic equation

$$x^3 + px^2 + qx + r = 0$$

where p, q and r are real, has roots α, β and γ.

(a) Given that

$$\alpha + \beta + \gamma = 4 \qquad \text{and} \qquad \alpha^2 + \beta^2 + \gamma^2 = 20$$

find the values of p and q. *(5 marks)*

(b) Given further that one root is $3 + \mathrm{i}$, find the value of r. *(5 marks)*

June 2006

5 The cubic equation

$$z^3 - 4\mathrm{i}z^2 + qz - (4 - 2\mathrm{i}) = 0$$

where q is a complex number, has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha + \beta + \gamma$; *(1 mark)*

(ii) $\alpha\beta\gamma$. *(1 mark)*

(b) Given that $\alpha = \beta + \gamma$, show that:

(i) $\alpha = 2\mathrm{i}$; *(1 mark)*

(ii) $\beta\gamma = -(1 + 2\mathrm{i})$; *(2 marks)*

(iii) $q = -(5 + 2\mathrm{i})$. *(3 marks)*

(c) Show that β and γ are the roots of the equation

$$z^2 - 2\mathrm{i}z - (1 + 2\mathrm{i}) = 0$$

(2 marks)

(d) Given that β is real, find β and γ. *(3 marks)*

February 2007

3 The cubic equation

$$z^3 + 2(1 - \mathrm{i})z^2 + 32(1 + \mathrm{i}) = 0$$

has roots α, β and γ.

(a) It is given that α is of the form $k\mathrm{i}$, where k is real. By substituting $z = k\mathrm{i}$ into the equation, show that $k = 4$. *(5 marks)*

(b) Given that $\beta = -4$, find the value of γ. *(2 marks)*

June 2007

2 The cubic equation

$$z^3 + pz^2 + 6z + q = 0$$

has roots α, β and γ.

(a) Write down the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. *(1 mark)*

(b) Given that p and q are real and that $\alpha^2 + \beta^2 + \gamma^2 = -12$:

(i) explain why the cubic equation has two non-real roots and one real root; *(2 marks)*

(ii) find the value of p. *(4 marks)*

(c) One root of the cubic equation is $-1 + 3\mathrm{i}$.

Find:

(i) the other two roots; *(3 marks)*

(ii) the value of q. *(2 marks)*

January 2008

4 The cubic equation

$$z^3 + iz^2 + 3z - (1 + i) = 0$$

has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha + \beta + \gamma$; *(1 mark)*

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$; *(1 mark)*

(iii) $\alpha\beta\gamma$. *(1 mark)*

(b) Find the value of:

(i) $\alpha^2 + \beta^2 + \gamma^2$; *(3 marks)*

(ii) $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$; *(4 marks)*

(iii) $\alpha^2\beta^2\gamma^2$. *(2 marks)*

(c) Hence write down a cubic equation whose roots are α^2, β^2 and γ^2. *(2 marks)*

June 2008

3 The cubic equation

$$z^3 + qz + (18 - 12i) = 0$$

where q is a complex number, has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha\beta\gamma$; *(1 mark)*

(ii) $\alpha + \beta + \gamma$. *(1 mark)*

(b) Given that $\beta + \gamma = 2$, find the value of:

(i) α; *(1 mark)*

(ii) $\beta\gamma$; *(2 marks)*

(iii) q. *(3 marks)*

(c) Given that β is of the form ki, where k is real, find β and γ. *(4 marks)*

January 2009

4 It is given that α, β and γ satisfy the equations

$$\alpha + \beta + \gamma = 1$$
$$\alpha^2 + \beta^2 + \gamma^2 = -5$$
$$\alpha^3 + \beta^3 + \gamma^3 = -23$$

(a) Show that $\alpha\beta + \beta\gamma + \gamma\alpha = 3$. *(3 marks)*

(b) Use the identity

$$(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) = \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$$

to find the value of $\alpha\beta\gamma$. *(2 marks)*

(c) Write down a cubic equation, with integer coefficients, whose roots are α, β and γ. *(2 marks)*

(d) Explain why this cubic equation has two non-real roots. *(2 marks)*

(e) Given that α is real, find the values of α, β and γ. *(4 marks)*

June 2009

3 The cubic equation

$$z^3 + pz^2 + 25z + q = 0$$

where p and q are real, has a root $\alpha = 2 - 3\mathrm{i}$.

(a) Write down another non-real root, β, of this equation. *(1 mark)*

(b) Find:

(i) the value of $\alpha\beta$; *(1 mark)*

(ii) the third root, γ, of the equation; *(3 marks)*

(iii) the values of p and q. *(3 marks)*

January 2010

3 The cubic equation

$$2z^3 + pz^2 + qz + 16 = 0$$

where p and q are real, has roots α, β and γ.

It is given that $\alpha = 2 + 2\sqrt{3}\,\mathrm{i}$.

(a) (i) Write down another root, β, of the equation. *(1 mark)*

(ii) Find the third root, γ. *(3 marks)*

(iii) Find the values of p and q. *(3 marks)*

June 2010

4 The roots of the cubic equation

$$z^3 - 2z^2 + pz + 10 = 0$$

are α, β and γ.

It is given that $\alpha^3 + \beta^3 + \gamma^3 = -4$.

(a) Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

(b) (i) Explain why $\alpha^3 - 2\alpha^2 + p\alpha + 10 = 0$. *(1 mark)*

(ii) Hence show that

$$\alpha^2 + \beta^2 + \gamma^2 = p + 13$$ *(4 marks)*

(iii) Deduce that $p = -3$. *(2 marks)*

(c) (i) Find the real root α of the cubic equation $z^3 - 2z^2 - 3z + 10 = 0$. *(2 marks)*

(ii) Find the values of β and γ. *(3 marks)*

January 2011

3 (a)		Show that $(1+\mathrm{i})^3 = 2\mathrm{i} - 2$.	*(2 marks)*
(b)		The cubic equation $$z^3 - (5+\mathrm{i})z^2 + (9+4\mathrm{i})z + k(1+\mathrm{i}) = 0$$ where k is a real constant, has roots α, β and γ. It is given that $\alpha = 1 + \mathrm{i}$.	
	(i)	Find the value of k.	*(3 marks)*
	(ii)	Show that $\beta + \gamma = 4$.	*(1 mark)*
	(iii)	Find the values of β and γ.	*(5 marks)*

June 2011

4		The cubic equation $$z^3 - 2z^2 + k = 0 \qquad (k \neq 0)$$ has roots α, β and γ.	
(a)	**(i)**	Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$.	*(2 marks)*
	(ii)	Show that $\alpha^2 + \beta^2 + \gamma^2 = 4$.	*(2 marks)*
	(iii)	Explain why $\alpha^3 - 2\alpha^2 + k = 0$.	*(1 mark)*
	(iv)	Show that $\alpha^3 + \beta^3 + \gamma^3 = 8 - 3k$.	*(2 marks)*
(b)		Given that $\alpha^4 + \beta^4 + \gamma^4 = 0$:	
	(i)	show that $k = 2$;	*(4 marks)*
	(ii)	find the value of $\alpha^5 + \beta^5 + \gamma^5$.	*(3 marks)*

January 2012

7 The numbers α, β and γ satisfy the equations

$$\alpha^2 + \beta^2 + \gamma^2 = -10 - 12\text{i}$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = 5 + 6\text{i}$$

(a) Show that $\alpha + \beta + \gamma = 0$. *(2 marks)*

(b) The numbers α, β and γ are also the roots of the equation

$$z^3 + pz^2 + qz + r = 0$$

Write down the value of p and the value of q. *(2 marks)*

(c) It is also given that $\alpha = 3\text{i}$.

(i) Find the value of r. *(3 marks)*

(ii) Show that β and γ are the roots of the equation

$$z^2 + 3\text{i}z - 4 + 6\text{i} = 0$$ *(2 marks)*

(iii) Given that β is real, find the values of β and γ. *(3 marks)*

June 2012

4 The cubic equation

$$z^3 + pz + q = 0$$

has roots α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

(ii) Express $\alpha\beta\gamma$ in terms of q. *(1 mark)*

(b) Show that

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$ *(3 marks)*

(c) Given that $\alpha = 4 + 7\text{i}$ and that p and q are real, find the values of:

(i) β and γ; *(2 marks)*

(ii) p and q. *(3 marks)*

(d) Find a cubic equation with integer coefficients which has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. *(3 marks)*

January 2013

4 The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$. *(2 marks)*

(ii) Hence find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. *(2 marks)*

(b) The value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is -4.

(i) Explain why α, β and γ cannot all be real. *(1 mark)*

(ii) By considering $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, find the possible values of k. *(4 marks)*

June 2013

5 The cubic equation

$$z^3 + pz^2 + qz + 37 - 36\mathrm{i} = 0$$

where p and q are constants, has three complex roots, α, β and γ.

It is given that $\beta = -2 + 3\mathrm{i}$ and $\gamma = 1 + 2\mathrm{i}$.

(a) (i) Write down the value of $\alpha\beta\gamma$. *(1 mark)*

(ii) Hence show that $(8 + \mathrm{i})\alpha = 37 - 36\mathrm{i}$. *(2 marks)*

(iii) Hence find α, giving your answer in the form $m + n\mathrm{i}$, where m and n are integers. *(3 marks)*

(b) Find the value of p. *(1 mark)*

(c) Find the value of the complex number q. *(2 marks)*