Core 3: Differentiation

Past Paper Questions:
2006 - 2013

Name:

January 2006

1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \frac{3x+1}{2x+1}$, show that $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$. *(3 marks)*

June 2006

2 (a) Find $\frac{dy}{dx}$ when $y = (3x-1)^{10}$. *(2 marks)*

5 (a) A curve has equation $y = e^{2x} - 10e^{x} + 12x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x-coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^{x} + 6 = 0$$ *(1 mark)*

(ii) By using the substitution $z = e^x$, or otherwise, show that the x-coordinates of P and Q are $\ln 2$ and $\ln 3$. *(3 marks)*

(iii) Find the y-coordinates of P and Q, giving each of your answers in the form $m + 12 \ln n$, where m and n are integers. *(3 marks)*

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. *(3 marks)*

January 2007

6 (a) Find $\frac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

(ii) $y = x^2 \tan x$. *(2 marks)*

(b) (i) Find $\frac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

June 2007

1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x+1)\ln x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x+1)\ln x$ at the point where $x = 1$. *(4 marks)*

2 (a) Differentiate $(x-1)^4$ with respect to x. *(1 mark)*

January 2008

1 (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; *(2 marks)*

(ii) $y = x\cos x$. *(2 marks)*

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. *(3 marks)*

June 2008

1 Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when:

(a) $y = (3x+1)^5$; *(2 marks)*

(b) $y = \ln(3x+1)$; *(2 marks)*

(c) $y = (3x+1)^5\ln(3x+1)$. *(3 marks)*

January 2009

6 A curve has equation $y = e^{2x}(x^2 - 4x - 2)$.

(a) Find the value of the x-coordinate of each of the stationary points of the curve. *(6 marks)*

(b) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(ii) Determine the nature of each of the stationary points of the curve. *(2 marks)*

June 2009 Question 1

(b) (i) Given that $y = \frac{\cos x}{2x + 1}$, use the quotient rule to find an expression for $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence find the gradient of the normal to the curve $y = \frac{\cos x}{2x + 1}$ at the point on the curve where $x = 0$. *(2 marks)*

January 2010

1 A curve has equation $y = e^{-4x}(x^2 + 2x - 2)$.

(a) Show that $\frac{dy}{dx} = 2e^{-4x}(5 - 3x - 2x^2)$. *(3 marks)*

(b) Find the exact values of the coordinates of the stationary points of the curve. *(5 marks)*

7 It is given that $y = \tan 4x$.

(a) By writing $\tan 4x$ as $\frac{\sin 4x}{\cos 4x}$, use the quotient rule to show that $\frac{dy}{dx} = p(1 + \tan^2 4x)$, where p is a number to be determined. *(3 marks)*

(b) Show that $\frac{d^2y}{dx^2} = qy(1 + y^2)$, where q is a number to be determined. *(5 marks)*

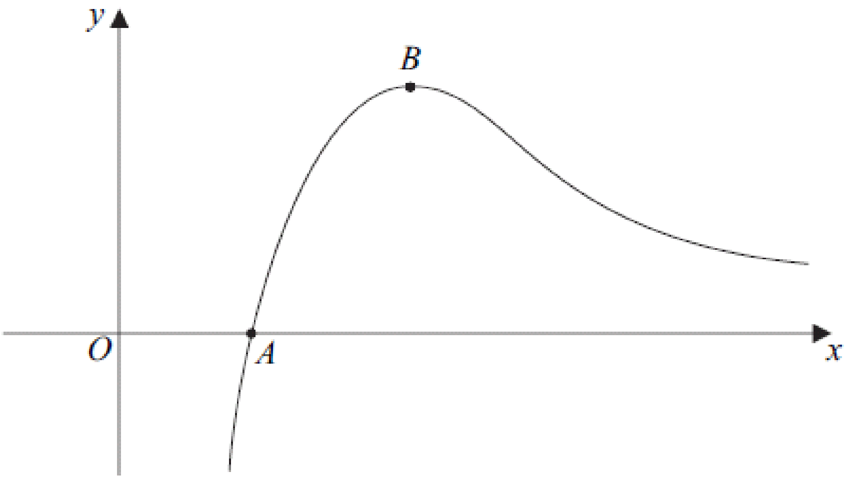
June 2010

3 (a) Find $\frac{dy}{dx}$ when:

(i) $y = \ln(5x - 2)$; *(2 marks)*

(ii) $y = \sin 2x$. *(2 marks)*

6 The diagram shows the curve $y = \frac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(a) State the coordinates of A. *(1 mark)*

(b) Find the coordinates of the stationary point, B, of the curve, giving your answer in an exact form. *(5 marks)*

(c) Find the exact value of the gradient of the normal to the curve at the point where $x = e^3$. *(3 marks)*

January 2011

1 (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$. *(2 marks)*

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$. *(3 marks)*

3 (a) Given that $x = \tan(3y + 1)$:

(i) find $\frac{dx}{dy}$ in terms of y; *(2 marks)*

(ii) find the value of $\frac{dy}{dx}$ when $y = -\frac{1}{3}$. *(2 marks)*

June 2011

2 (a) (i) Find $\frac{dy}{dx}$ when $y = xe^{2x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. *(2 marks)*

(b) Given that $y = \frac{2\sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where k is an integer. *(4 marks)*

January 2012

6 (a) Given that $x = \frac{1}{\sin\theta}$, use the quotient rule to show that $\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$. *(3 marks)*

7 (a) A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates. *(7 marks)*

June 2012

3 A curve has equation $y = x^3 \ln x$.

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = e$. *(3 marks)*

(ii) This tangent intersects the x-axis at the point A. Find the exact value of the x-coordinate of the point A. *(2 marks)*

January 2013

3 (a) Find $\frac{dy}{dx}$ when

$$y = e^{3x} + \ln x$$ *(2 marks)*

(b) (i) Given that $u = \frac{\sin x}{1 + \cos x}$, show that $\frac{du}{dx} = \frac{1}{1 + \cos x}$. *(3 marks)*

(ii) Hence show that if $y = \ln\left(\frac{\sin x}{1 + \cos x}\right)$, then $\frac{dy}{dx} = \operatorname{cosec} x$. *(2 marks)*

June 2013

2 (a) Given that $y = x^4 \tan 2x$, find $\frac{dy}{dx}$. *(3 marks)*

(b) Find the gradient of the curve with equation $y = \frac{x^2}{x - 1}$ at the point where $x = 3$.
(3 marks)