D2: Simplex Algorithm

Past Paper Questions
2006 - 2013

Name:

January 2006

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 2y + 4z$

subject to $x + 4y + 2z \leqslant 8$
$2x + 7y + 3z \leqslant 21$
$x \geqslant 0, y \geqslant 0, z \geqslant 0$ *(3 marks)*

(b) Use the Simplex method to perform **one** iteration of your tableau for part (a), choosing a value in the z-column as pivot. *(3 marks)*

(c) (i) Perform one further iteration. *(5 marks)*

(ii) State whether or not this is the optimal solution, and give a reason for your answer. *(2 marks)*

June 2006

5 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 4x + 9y$. The initial Simplex tableau is given below.

P	***x***	***y***	***r***	***s***	***t***	***value***
1	−4	−9	0	0	0	0
0	3	7	1	0	0	33
0	1	2	0	1	0	10
0	2	7	0	0	1	26

(a) Write down the **three** inequalities in x and y represented by this tableau. *(2 marks)*

(b) The Simplex method is to be used to solve this linear programming problem by initially choosing a value in the x-column as the pivot.

(i) Explain why the initial pivot has value 1. *(2 marks)*

(ii) Perform **two** iterations using the Simplex method. *(7 marks)*

(iii) Comment on how you know that the optimum solution has been achieved and state your final values of P, x and y. *(3 marks)*

January 2007

3 (a) Display the following linear programming problem in a Simplex tableau.

$$\begin{aligned} &\text{Maximise} && P = 5x + 8y + 7z \\ &\text{subject to} && 3x + 2y + z \leqslant 12 \\ & && 2x + 4y + 5z \leqslant 16 \\ & && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0 \end{aligned}$$

(3 marks)

(b) The Simplex method is to be used by initially choosing a value in the y-column as a pivot.

(i) Explain why the initial pivot is 4. *(1 mark)*

(ii) Perform **two** iterations of your tableau from part (a) using the Simplex method. *(6 marks)*

(iii) State the values of P, x, y and z after your second iteration. *(2 marks)*

(iv) State, giving a reason, whether the maximum value of P has been achieved. *(1 mark)*

June 2007

4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 5y$. The initial Simplex tableau is given below.

P	***x***	***y***	***s***	***t***	***u***	***value***
1	−3	−5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, write down **three** inequalities involving x and y for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the ***y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

January 2008

4 A linear programming problem involving the variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 3y + 5z$. The initial Simplex tableau is given below.

P	*x*	*y*	*z*	*s*	*t*	*u*	*value*
1	−2	−3	−5	0	0	0	0
0	1	0	1	1	0	0	9
0	2	1	4	0	1	0	40
0	4	2	3	0	0	1	33

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the z-column, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

June 2008

4 A linear programming problem consists of maximising an objective function P involving three variables x, y and z. Slack variables s, t, u and v are introduced and the Simplex method is used to solve the problem. Several iterations of the method lead to the following tableau.

P	*x*	*y*	*z*	*s*	*t*	*u*	*v*	*value*
1	0	−12	0	5	−3	0	0	37
0	1	−8	0	1	2	0	0	16
0	0	4	0	0	3	0	1	20
0	0	2	0	−3	2	1	0	14
0	0	1	1	2	5	0	0	8

(a) (i) The pivot for the next iteration is chosen from the ***y*-column**. State which value should be chosen and explain the reason for your choice. *(2 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) Explain why your new tableau solves the original problem. *(1 mark)*

(c) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(d) State the values of the slack variables at the optimum point. Hence determine how many of the original inequalities still have some slack when the optimum is reached. *(2 marks)*

3 (a) Display the following linear programming problem in a Simplex tableau.

$$\begin{aligned} \text{Maximise} \quad & P = 4x - 5y + 6z \\ \text{subject to} \quad & 6x + 7y - 4z \leqslant 30 \\ & 2x + 4y - 5z \leqslant 8 \\ & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0 \end{aligned}$$

(3 marks)

(b) The Simplex method is to be used to solve this problem.

(i) Explain why it is not possible to choose a pivot from the z-column initially. *(1 mark)*

(ii) Identify the initial pivot and explain why this particular element should be chosen. *(2 marks)*

(iii) Perform one iteration using your initial tableau from part (a). *(3 marks)*

(iv) State the values of x, y and z after this first iteration. *(2 marks)*

(v) Without performing any further iterations, explain why P has no finite maximum value. *(1 mark)*

(c) Using the same inequalities as in part (a), the problem is modified to

$$\text{Maximise} \quad Q = 4x - 5y - 20z$$

(i) Write down a modified initial tableau and the revised tableau after one iteration. *(2 marks)*

(ii) Hence find the maximum value of Q. *(1 mark)*

June 2009

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 4x + y + kz$, where k is a constant. The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***value***
1	−4	−1	$-k$	0	0	0
0	1	2	3	1	0	7
0	2	1	4	0	1	10

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, write down **two** inequalities involving x, y and z for this problem. *(1 mark)*

(b) (i) The first pivot is chosen from the ***x*-column**. Identify the pivot and perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value of P has not been reached after this first iteration, find the possible values of k. *(2 marks)*

(c) Given that $k = 10$:

(i) perform one further iteration of the Simplex method; *(4 marks)*

(ii) interpret the final tableau. *(3 marks)*

January 2010

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 4y + 3z$. The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−2	−4	−3	0	0	0	0
0	2	2	1	1	0	0	14
0	−1	1	2	0	1	0	6
0	4	4	3	0	0	1	29

(a) (i) What name is given to the variables s, t and u? *(1 mark)*

(ii) Write down an equation involving x, y, z and s for this problem. *(1 mark)*

(b) (i) By choosing the first pivot **from the *y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau, stating the values of P, x, y and z. *(3 marks)*

June 2010

3 (a) Given that k is a constant, display the following linear programming problem in a Simplex tableau.

$$\begin{aligned} &\text{Maximise} && P = 6x + 5y + 3z \\ &\text{subject to} && x + 2y + kz \leqslant 8 \\ & && 2x + 10y + z \leqslant 17 \\ & && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0 \end{aligned}$$

(3 marks)

(b) (i) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the x-column as pivot. *(4 marks)*

(ii) Given that the maximum value of P has not been achieved after this first iteration, find the range of possible values of k. *(2 marks)*

(c) In the case where $k = -1$, perform one further iteration and interpret your final tableau. *(6 marks)*

January 2011

4 The Simplex method is to be used to maximise $P = 3x + 2y + z$ subject to the constraints

$$-x + y + z \leqslant 4$$

$$2x + y + 4z \leqslant 10$$

$$4x + 2y + 3z \leqslant 21$$

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−3	−2	−1	0	0	0	0
0	−1	1	1	1	0	0	4
0	2	1	4	0	1	0	10
0	4	2	3	0	0	1	21

(a) (i) The first pivot is to be chosen from the x-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method and explain how you know that the optimal value has not been reached. *(5 marks)*

(b) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and write down the initial inequality that still has slack. *(4 marks)*

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 6y + kz$, where k is a constant.

The initial Simplex tableau is given below.

P	*x*	*y*	*z*	*s*	*t*	*u*	*value*
1	−2	−6	$-k$	0	0	0	0
0	5	3	10	1	0	0	15
0	7	6	4	0	1	0	28
0	4	3	6	0	0	1	12

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot **from the *y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value has **not** been reached, find the possible values of k. *(2 marks)*

(c) In the case when $k = 20$:

(i) perform one further iteration; *(4 marks)*

(ii) interpret the final tableau and state the values of the slack variables. *(3 marks)*

4 A linear programming problem consists of maximising an objective function P involving three variables, x, y and z, subject to constraints given by three inequalities other than $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$. Slack variables s, t and u are introduced and the Simplex method is used to solve the problem. One iteration of the method leads to the following tableau.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−2	11	0	3	0	0	6
0	2	3	1	1	0	0	2
0	6	−30	0	−6	1	0	3
0	−1	−9	0	−3	0	1	4

(a) (i) State the column from which the pivot for the **next** iteration should be chosen. Identify this pivot and explain the reason for your choice. *(3 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) (i) Explain why you know that the maximum value of P has been achieved. *(1 mark)*

(ii) State how many of the three original inequalities still have slack. *(1 mark)*

(c) (i) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(ii) The objective function for this problem is $P = kx - 2y + 3z$, where k is a constant. Find the value of k. *(2 marks)*

3 (a) Given that k is a constant, complete the Simplex tableau below for the following linear programming problem.

Maximise $P = kx + 6y + 5z$

subject to $2x + y + 4z \leqslant 11$
$x + 3y + 6z \leqslant 18$
$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$ *(2 marks)*

(b) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the ***y*-column** as pivot. *(4 marks)*

(c) (i) In the case when $k = 1$, explain why the maximum value of P has now been reached and write down this maximum value of P. *(2 marks)*

(ii) In the case when $k = 3$, perform one further iteration and interpret your new tableau. *(6 marks)*

(a)

P	*x*	*y*	*z*	*s*	*t*	*value*
1	$-k$	-6	-5	0	0	0
0						
0						

(b)

P	*x*	*y*	*z*	*s*	*t*	*value*

(c)(ii)

P	*x*	*y*	*z*	*s*	*t*	*value*

January 2013

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = x - 2y + 3z$

subject to
$$x + y + z \leqslant 16$$
$$x - 2y + 2z \leqslant 17$$
$$2x - y + 2z \leqslant 19$$

and $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$. *(2 marks)*

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret the tableau that you obtained in part **(c)(i)** and state the values of your slack variables. *(3 marks)*

June 2013

6 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x + 3y + z$

subject to
$$2x + y + z \leqslant 25$$
$$x + 2y + z \leqslant 40$$
$$x + y + 2z \leqslant 30$$

and $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$. *(2 marks)*

(b) The first pivot to be chosen is from the x-column.

Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret your final tableau and state the values of your slack variables. *(3 marks)*