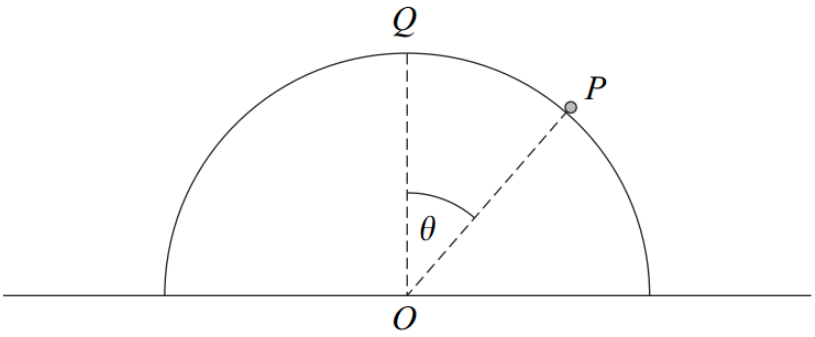
D2: Motion in a vertical circle

Past Paper Questions
2006 - 2013

Name:

January 2006

7 A particle P, of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O. The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of $2\,\mathrm{m\,s^{-1}}$. When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v\,\mathrm{m\,s^{-1}}$.

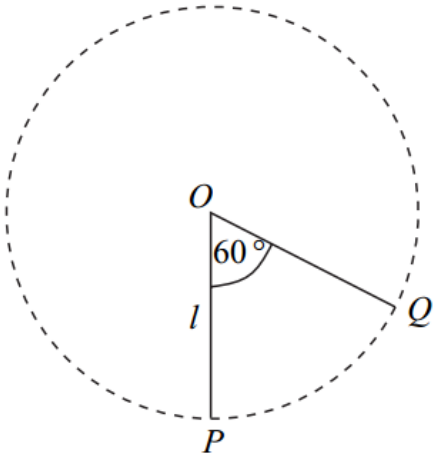


(a) Show that $v^2 = 4 + 6g(1 - \cos\theta)$. *(4 marks)*

(b) Find the value of θ when the particle leaves the hemisphere. *(5 marks)*

June 2006

4 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l. The particle hangs in equilibrium at the point P vertically below O. The particle is then set into motion with a horizontal velocity U so that it moves in a complete vertical circle with centre O. The point Q on the circle is such that $\angle POQ = 60°$, as shown in the diagram.



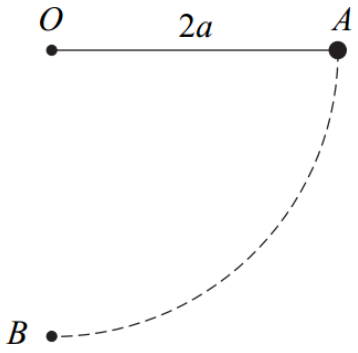
(a) Find, in terms of g, l and U, the speed of the particle at Q. *(4 marks)*

(b) Find, in terms of g, l, m and U, the tension in the string when the particle is at Q. *(5 marks)*

(c) Find, in terms of g, l, m and U, the tension in the string when the particle returns to P. *(2 marks)*

January 2007

3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

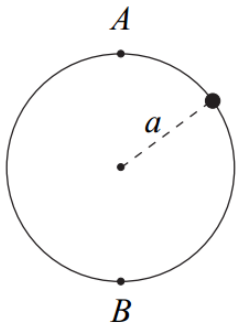


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

June 2007

5 A bead of mass m moves on a smooth circular ring of radius a which is fixed in a vertical plane, as shown in the diagram. Its speed at A, the highest point of its path, is v and its speed at B, the lowest point of its path, is $7v$.

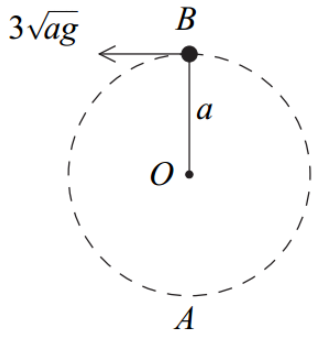


(a) Show that $v = \sqrt{\frac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of m and g, when the bead is at A. *(4 marks)*

January 2008

7 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end. The particle is moving in a vertical circle, centre O. When the particle is at B, vertically above O, the string is taut and the particle is moving with speed $3\sqrt{ag}$.

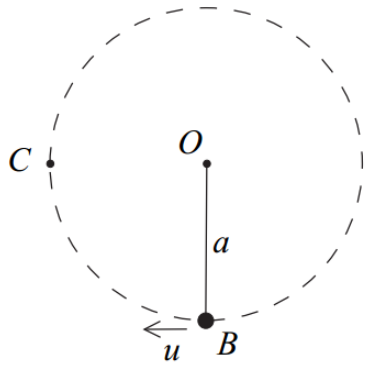


(a) Find, in terms of g and a, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of g and m, the tension in the string when the particle is at A. *(4 marks)*

June 2008

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.

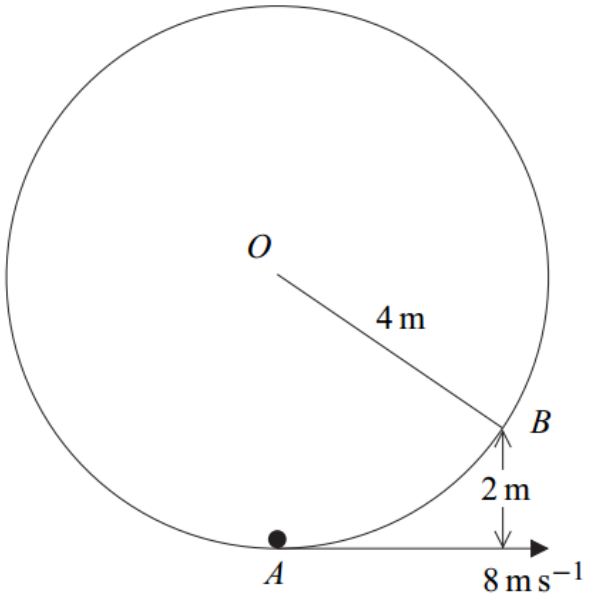


(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

7 A hollow cylinder, of internal radius 4 m, is fixed so that its axis is horizontal. The point O is on this axis. A particle, of mass 6 kg, is set in motion so that it moves on the smooth **inner** surface of the cylinder in a vertical circle about O. Its speed at the point A, which is vertically below O, is $8\,\text{m}\,\text{s}^{-1}$.



When the particle is at the point B, at a height of 2 m above A, find:

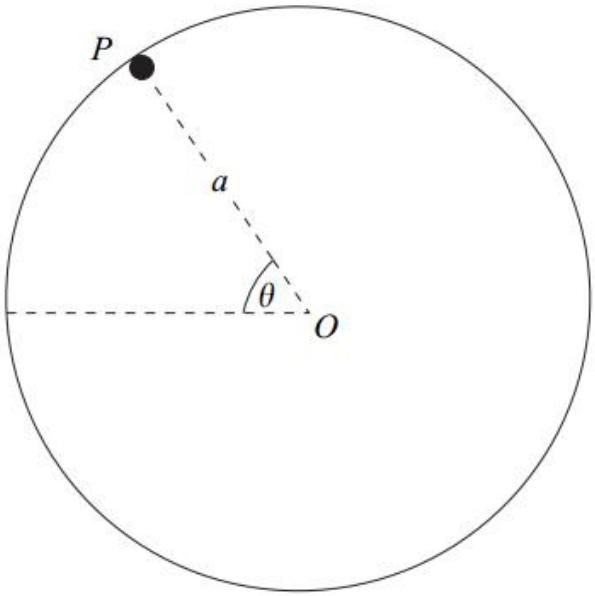
(a) its speed; *(3 marks)*

(b) the normal reaction between the cylinder and the particle. *(4 marks)*

7 In crazy golf, a golf ball is hit so that it starts to move in a vertical circle on the inside of a smooth cylinder.

Model the golf ball as a particle, P, of mass m. The circular path of the golf ball has radius a and centre O. At time t, the angle between OP and the horizontal is θ, as shown in the diagram.

The golf ball has speed u at the lowest point of its circular path.



(a) Show that, while the golf ball is in contact with the cylinder, the reaction of the cylinder on the golf ball is

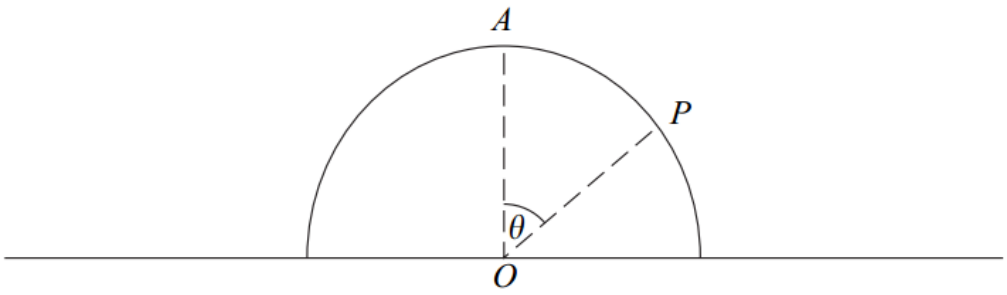
$$\frac{mu^2}{a} - 3mg\sin\theta - 2mg$$ *(6 marks)*

(b) Given that $u = \sqrt{3ag}$, the golf ball will not complete a vertical circle inside the cylinder. Find the angle which OP makes with the horizontal when the golf ball leaves the surface of the cylinder. *(4 marks)*

7 A smooth hemisphere, of radius a and centre O, is fixed with its plane face on a horizontal surface. A particle, of mass m, can move freely on the surface of the hemisphere.

The particle is placed at the point A, the highest point of the hemisphere, and is set in motion along the surface with speed u.

(a) While the particle is in contact with the hemisphere at a point P, OP makes an angle θ with the upward vertical.



Show that the speed of the particle at P is

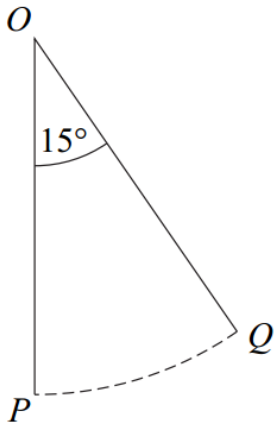
$$(u^2 + 2ga[1 - \cos\theta])^{\frac{1}{2}}$$ *(5 marks)*

(b) The particle leaves the surface of the hemisphere when $\theta = \alpha$.

Find $\cos\alpha$ in terms of a, u and g. *(5 marks)*

June 2010

8 A particle is attached to one end of a light inextensible string of length 3 metres. The other end of the string is attached to a fixed point O. The particle is set into motion horizontally at point P with speed v, so that it describes part of a vertical circle whose centre is O. The point P is vertically below O.



The particle first comes momentarily to rest at the point Q, where OQ makes an angle of 15° to the vertical.

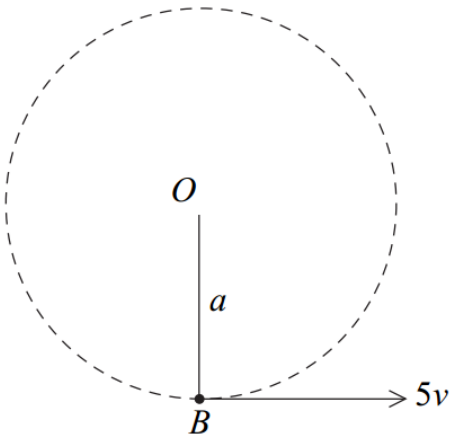
(a) Find the value of v. *(4 marks)*

(b) When the particle is at rest at the point Q, the tension in the string is 22 newtons.

Find the mass of the particle. *(3 marks)*

January 2011

6 A light inextensible string, of length a, has one end attached to a fixed point O. A small bead, of mass m, is attached to the other end of the string. The bead is moving in a vertical circle, centre O. When the bead is at B, vertically below O, the string is taut and the bead is moving with speed $5v$.

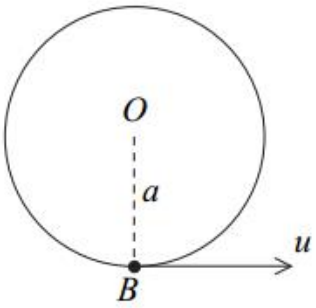


(a) The speed of the bead at the highest point of its path is $3v$.

Find v in terms of a and g. *(4 marks)*

(b) Find the ratio of the greatest tension to the least tension in the string, as the bead travels around its circular path. *(5 marks)*

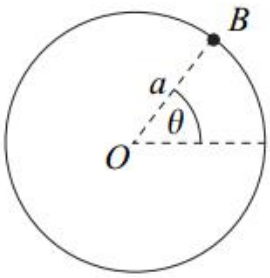
8 A smooth wire is fixed in a vertical plane so that it forms a circle of radius a metres and centre O. A bead, B, of mass 0.3 kg, is threaded on the wire and is set in motion with a speed $u\,\mathrm{m\,s^{-1}}$ at the lowest point of its circular path, as shown in the diagram.



(a) Show that, if the bead is going to make complete revolutions around the wire,

$$u > 2\sqrt{ag}$$ *(3 marks)*

(b) At time t seconds, the angle between OB and the horizontal is θ, as shown in the diagram.

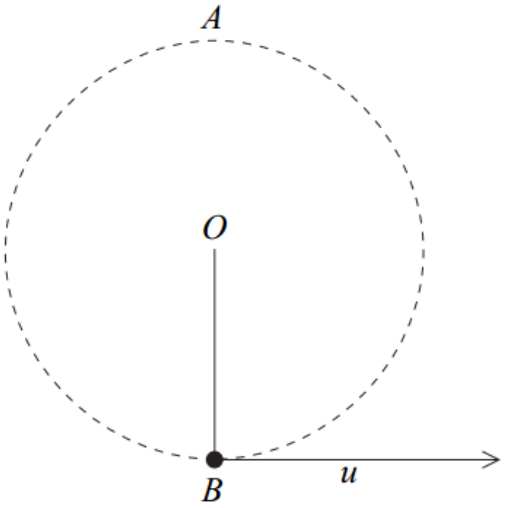


It is given that $u = \sqrt{\frac{9}{2}ag}$.

(i) Find the reaction of the bead on the wire, giving your answer in terms of g and θ. *(5 marks)*

(ii) Find θ when this reaction is zero. *(2 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string of length a. With the string taut, the bead is at the point B, vertically below O, when it is set into vertical circular motion with an initial horizontal velocity u, as shown in the diagram.



The string does not become slack in the subsequent motion. The velocity of the bead at the point A, where A is vertically above O, is v.

(a) Show that $v^2 = u^2 - 4ag$. *(2 marks)*

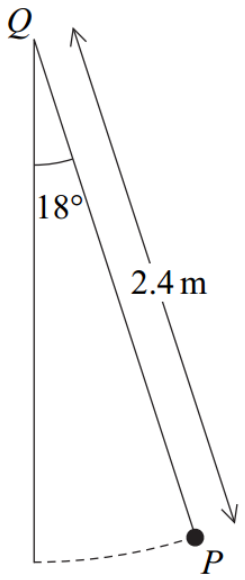
(b) The ratio of the tensions in the string when the bead is at the two points A and B is 2:5.

(i) Find u in terms of g and a. *(7 marks)*

(ii) Find the ratio $u:v$. *(2 marks)*

June 2012

6 Simon, a small child of mass 22 kg, is on a swing. He is swinging freely through an angle of 18° on both sides of the vertical. Model Simon as a particle, P, of mass 22 kg, attached to a fixed point, Q, by a light inextensible rope of length 2.4 m.



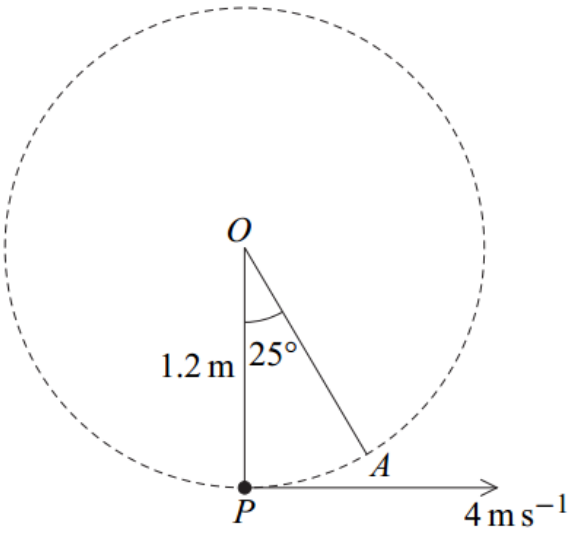
(a) Find Simon's maximum speed as he swings. *(4 marks)*

(b) Calculate the tension in the rope when Simon's speed is a maximum. *(3 marks)*

January 2013

7 A small ball, of mass 3 kg, is suspended from a fixed point O by a light inextensible string of length 1.2 m. Initially, the string is taut and the ball is at the point P, vertically below O. The ball is then set into motion with an initial horizontal velocity of $4\,\text{m s}^{-1}$.

The ball moves in a vertical circle, centre O. The point A, on the circle, is such that angle AOP is 25°, as shown in the diagram.



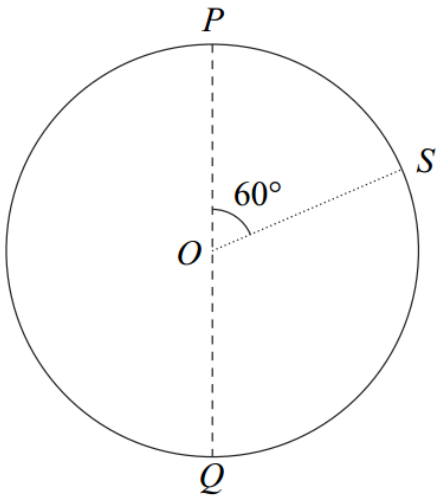
(a) Find the speed of the ball at the point A. *(4 marks)*

(b) Find the tension in the string when the ball is at the point A. *(3 marks)*

8 A bead, of mass m, moves on a smooth circular ring, of radius a and centre O, which is fixed in a vertical plane. At P, the highest point on the ring, the speed of the bead is $2u$; at Q, the lowest point on the ring, the speed of the bead is $5u$.

(a) Show that $u = \sqrt{\frac{4ag}{21}}$. *(4 marks)*

(b) S is a point on the ring so that angle POS is 60°, as shown in the diagram.



Find, in terms of m and g, the magnitude of the reaction of the ring on the bead when the bead is at S. *(5 marks)*