D2: Dynamic Programming

Past Paper Questions
2006 - 2013

Name:

January 2006

2 A manufacturing company is planning to build three new machines, A, B and C, at the rate of one per month. The order in which they are built is a matter of choice, but the profits will vary according to the number of workers available and the suppliers' costs. The expected profits in thousands of pounds are given in the table.

Month	Already built	Profit (in units of £1000)		
		A	***B***	***C***
1	—	52	47	48
2	***A***	—	58	54
	B	70	—	54
	C	68	63	—
3	***A* and *B***	—	—	64
	A* and *C	—	67	—
	B* and *C	69	—	—

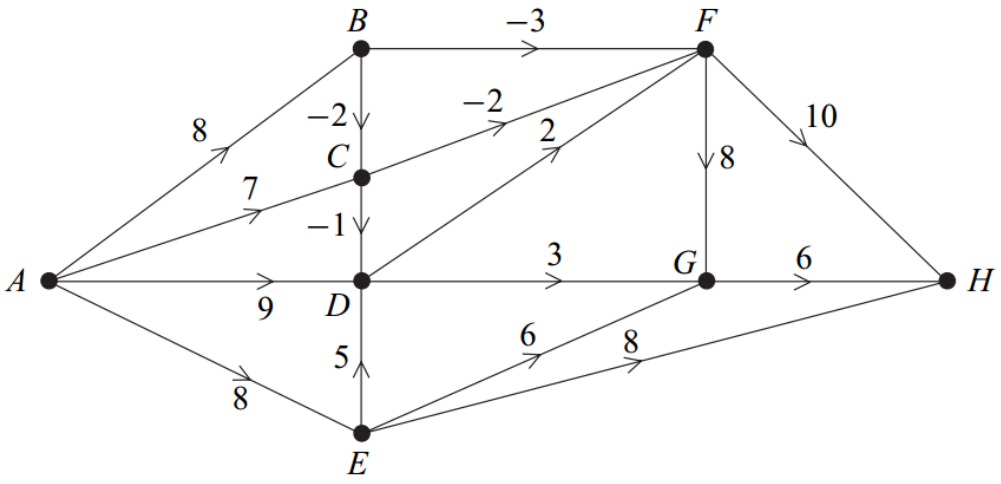
(a) Draw a labelled network such that the most profitable order of manufacture corresponds to the longest path within that network. *(2 marks)*

(b) Use dynamic programming to determine the order of manufacture that **maximises** the total profit, and state this maximum profit. *(7 marks)*

June 2006

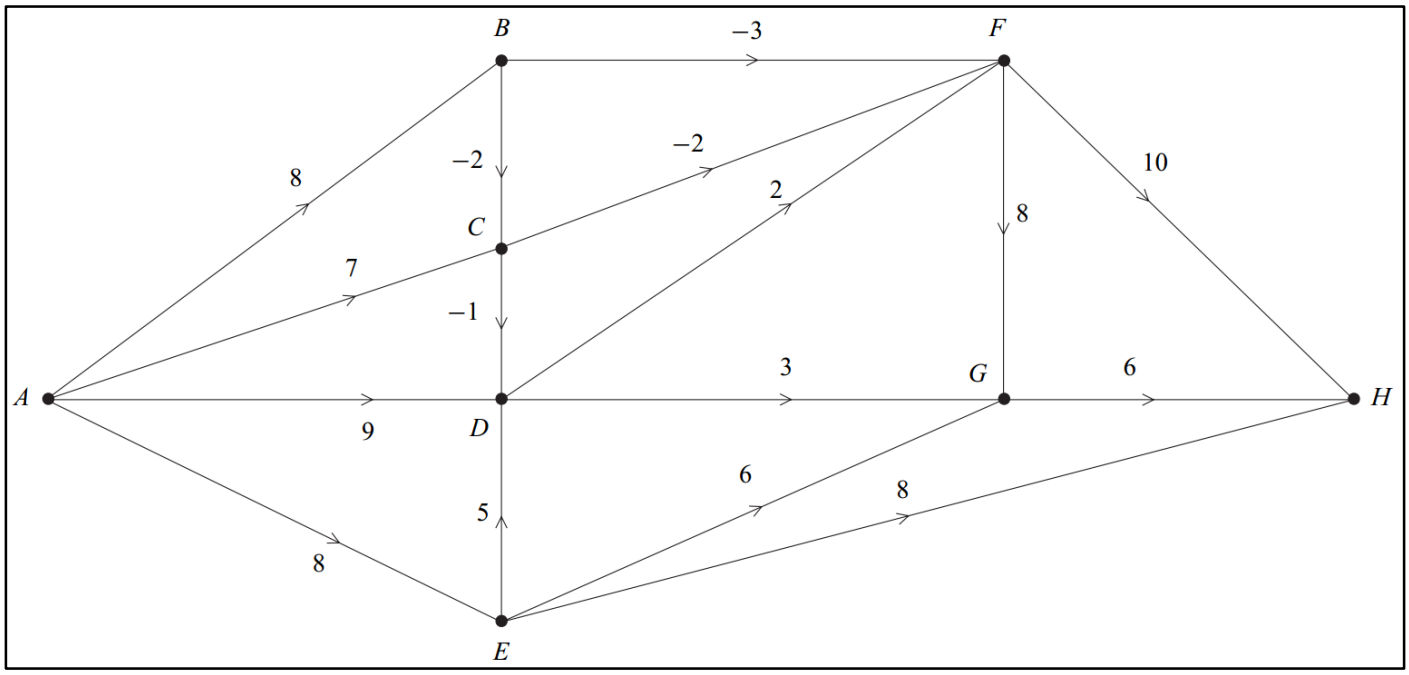
3 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows eight vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



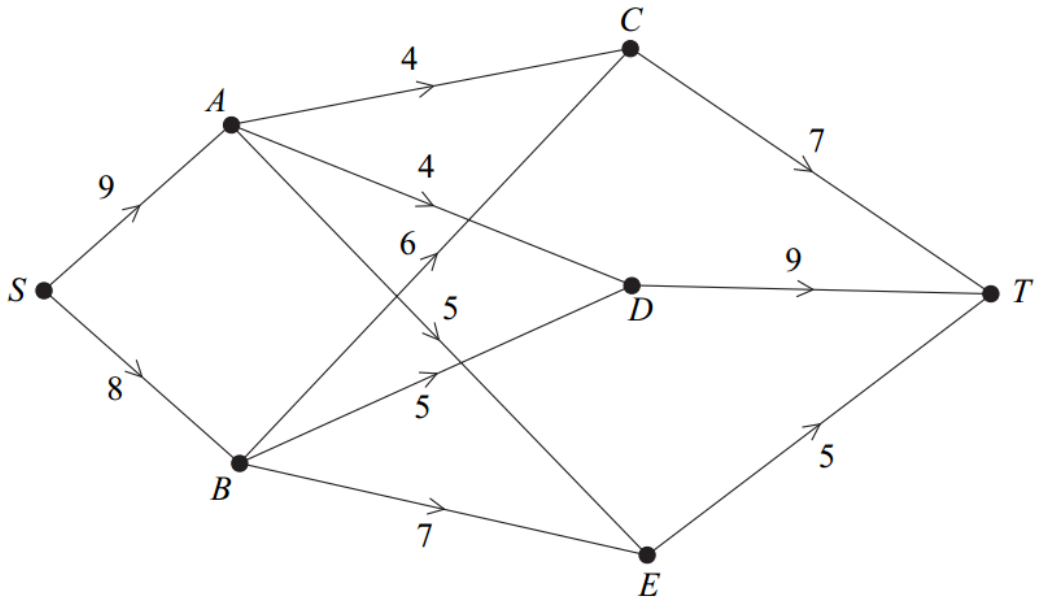
(a) Use dynamic programming to find the minimum cost of travelling from A to H. You may use **Figure 3** for your working. *(6 marks)*

(b) State the minimum cost and the possible routes corresponding to this minimum cost. *(3 marks)*



January 2007

5 A three-day journey is to be made from S to T, with overnight stops at the end of the first day at either A or B and at the end of the second day at one of the locations C, D or E. The network shows the number of hours of sunshine forecast for each day of the journey.



The optimal route, known as the maximin route, is that for which the least number of hours of sunshine during a day's journey is as large as possible.

(a) Explain why the three-day route $SAET$ is better than $SACT$. *(2 marks)*

(b) Use dynamic programming to find the optimal (maximin) three-day route from S to T. *(8 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A maker of exclusive furniture is planning to build three cabinets A, B and C at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

Month	**Already built**	**Cost**		
		A	***B***	***C***
1	–	500	440	475
2	***A***	–	440	490
	B	510	–	500
	C	520	490	–
3	***A* and *B***	–	–	520
	A* and *C	–	500	–
	B* and *C	510	–	–

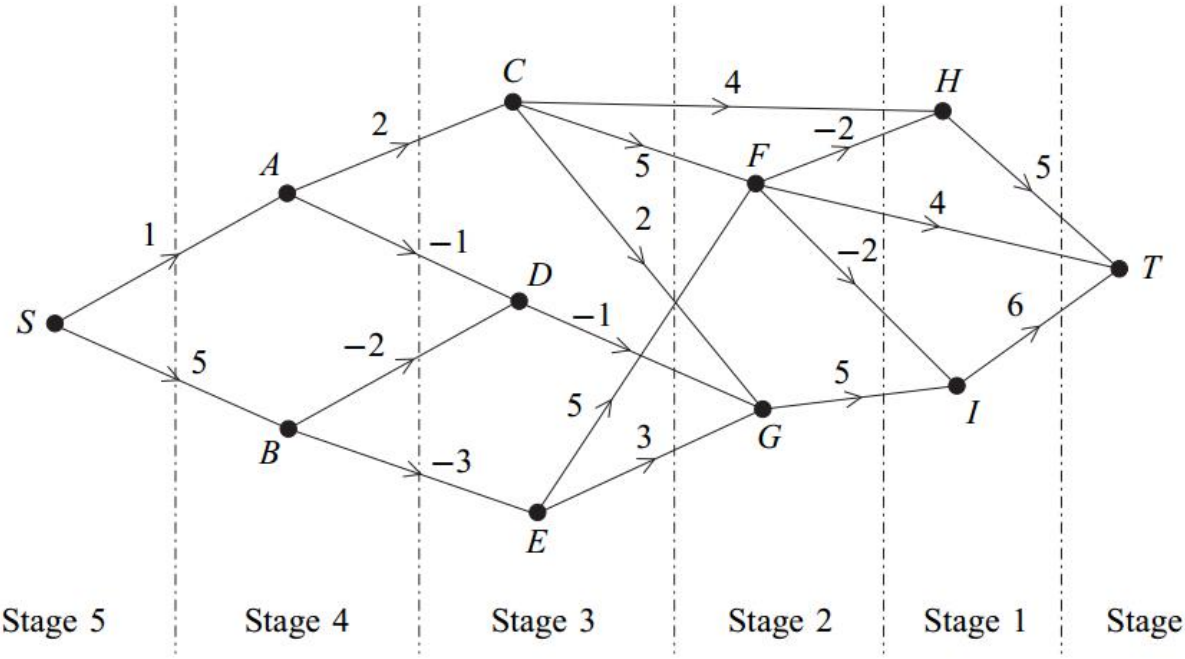
(a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. *(6 marks)*

(b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. *(4 marks)*

Month	Already built	Machine built	For use in part (a)		For use in part (b)
3	*A* and *B*	*C*			
	A and *C*	*B*			
	B and *C*	*A*			
2	*A*	*B*			
		C			
	B	*A*			
		C			
	C	*A*			
		B			

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows 11 vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



(a) **Working backwards from *T***, use dynamic programming to find the minimum cost of travelling from *S* to *T*. You may wish to complete the table on **Figure 3** as your solution. *(6 marks)*

(b) State the minimum cost and the routes corresponding to this minimum cost. *(3 marks)*

Stage	State	From	Value	
1	H	T	5	
	I	T	6	
2	F	H	$-2+5=3$	
		T	4	
		I		
	G	I		
3	C	H		
		F		
		G		

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A small firm produces high quality cabinets.

It can produce up to 4 cabinets each month.

Whenever at least one cabinet is made during that month, the overhead costs for that month are £300.

It is possible to hold in stock a maximum of 2 cabinets during any month.

The cost of storage is £50 per cabinet per month.

The orders for cabinets are shown in the table below. There is no stock at the beginning of January and the firm plans to clear all stock after completing the April orders.

Month	January	February	March	April
Number of cabinets required	3	3	5	2

(a) Determine the total cost of storing 2 cabinets and producing 3 cabinets in a given month. *(2 marks)*

(b) By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from April**, to find the production schedule which minimises total costs. *(8 marks)*

(c) Each cabinet is sold for £2000 but there is an additional cost of £300 for materials to make each cabinet and £2000 per month in wages. Determine the total profit for the four-month period. *(3 marks)*

Initial State is the number of cabinets in stock at the beginning of that month.

Action is the number of cabinets made during that month.

Destination State is the number of cabinets in stock at the end of that month after the demand has been met.

The destination state for March becomes the initial state for April and so on.

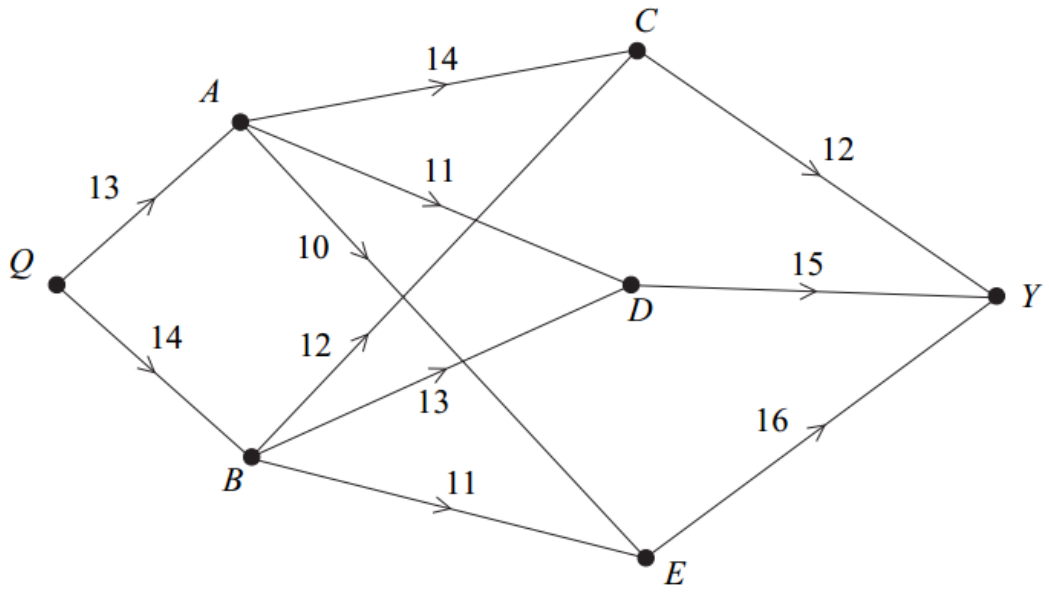
Month & Demand	Initial State	Action	Destination State	Value
April	0	2	0	$300 + 0 = 300$
(demand 2)				
	1	1	0	$300 + 50 = 350$
	2	0	0	$0 + 100 = 100$
March	1	4	0	$300 + 50 + 300 = 650$
(demand 5)				
	2	3	0	
		4	1	
February	0	4	1	
(demand 3)				
	1	3	1	
		4	2	
	2	2	1	
		3	2	
January	0	3	0	
(demand 3)		4	1	

Production Schedule which minimises total costs

Month	January	February	March	April
Number of cabinets made				

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A truck has to transport stones from a quarry, Q, to a builders yard, Y. The network shows the possible roads from Q to Y. Along each road there are bridges with weight restrictions. The value on each edge indicates the maximum load in tonnes that can be carried by the truck along that particular road.



The truck is able to carry a load of up to 20 tonnes. The optimal route, known as the maximin route, is that for which the possible load that the truck can carry is as large as possible.

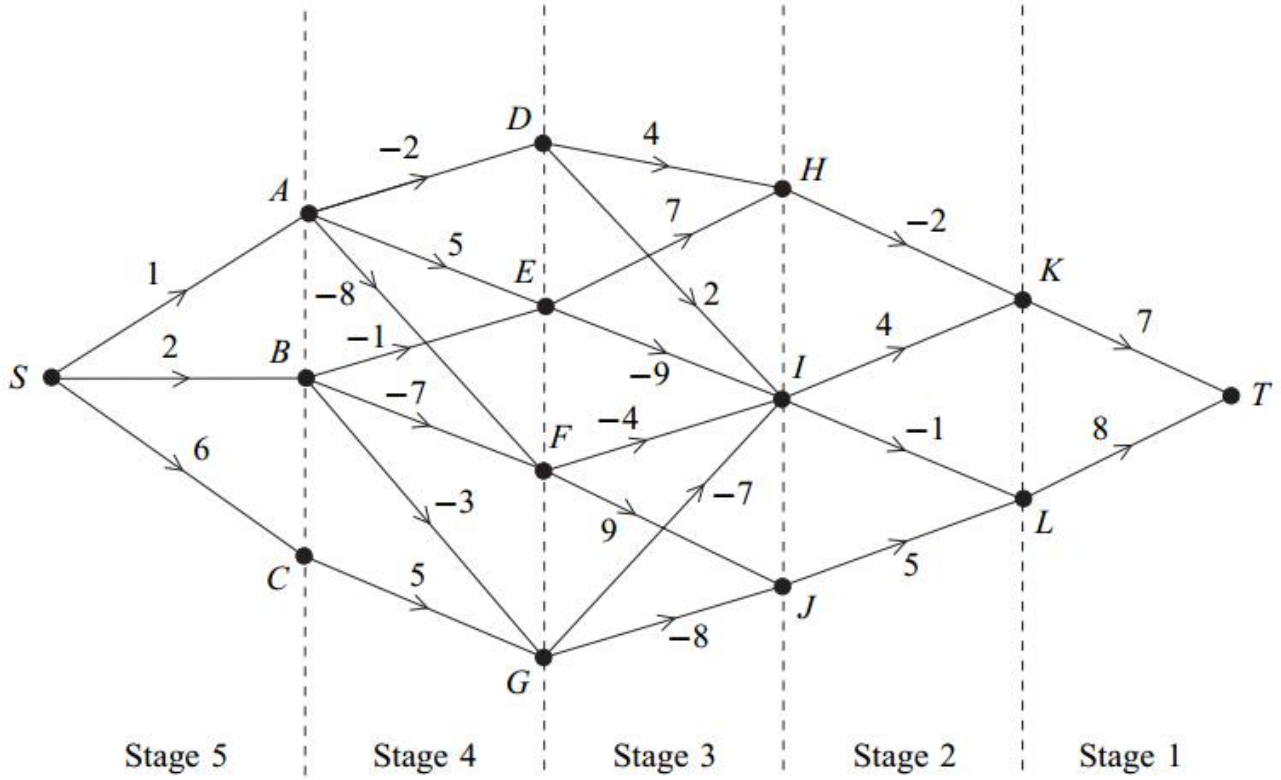
(a) Explain why the route $QACY$ is better than the route $QBEY$. *(2 marks)*

(b) By completing the table on **Figure 3**, or otherwise, use dynamic programming, **working backwards from *Y***, to find the optimal (maximin) route from Q to Y. Write down the maximin route and state the maximum possible load that the truck can carry from Q to Y. *(8 marks)*

Stage	State	Action	Calculation	Value
1	*C*	*CY*	–	12
	D	*DY*	–	
	E	*EY*	–	
2	*A*	*AC*	Min(14,12)	12

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

A company has a number of stores. The following network shows the possible actions and profits over the next five years. The number on each edge is the expected profit, in millions of pounds. A negative number indicates a loss due to investment in new stores.



(a) **Working backwards from *T***, use dynamic programming to maximise the expected profits over the five years. You may wish to complete the table on **Figure 2** as your solution. *(7 marks)*

(b) State the maximum expected profit and the sequence of vertices from S to T in order to achieve this. *(2 marks)*

Stage	State	From	Value	
1	K	T	7	
	L	T	8	
2	H	K	$-2+7=5$	
	I	K		
		L		
	J	L		
3	D	H		
		I		

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A landscape gardener has three projects, *A*, *B* and *C*, to be completed over a period of 4 months: May, June, July and August. The gardener must allocate one of these months to each project and the other month is to be taken as a holiday. Various factors, such as availability of materials and transport, mean that the costs for completing the projects in different months will vary. The costs, in thousands of pounds, are given in the table.

	May	**June**	**July**	**August**
Project *A*	17	16	18	16
Project *B*	14	13	12	10
Project *C*	14	17	15	14

By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from August**, to find the project schedule that minimises total costs. State clearly which month should be taken as a holiday and which project should be undertaken in which month. *(10 marks)*

(*0* in the Action column is used to represent no projects.)

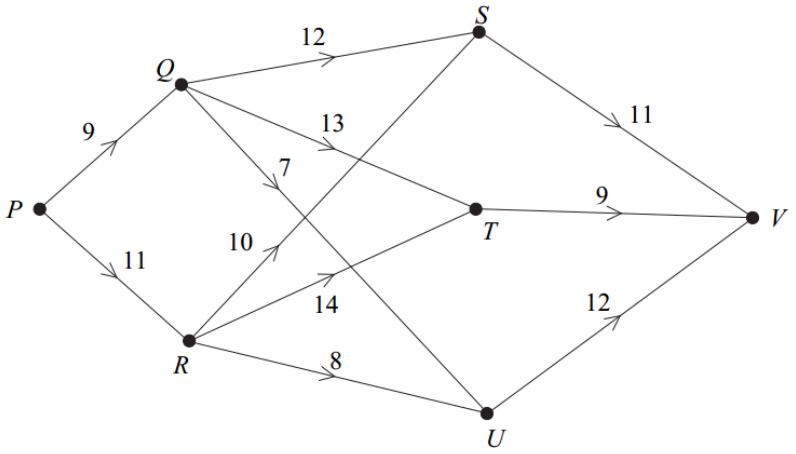
Stage (month)	State (projects already done)	Action (project to do)	Calculation	Cost in thousands of pounds
August	*A*, *B*, *C*	*0*		0
	A, *B*	*C*		14
	A, *C*	*B*		10
	B, *C*	*A*		16
July	*A*, *B*	*0*	$0 + 14$	14
		C	$15 + 0$	15
	A, *C*	*0*		
		B		
	B, *C*	*0*		
		A		
	A	*B*	$12 + 14$	26
		C		
	B	*A*		
		C		
	C	*A*		
		B		
June	*A*	*0*		
		B		
		C		
May				

Schedule

	May	June	July	August
Project				

June 2010

5 A three-day journey is to be made from P to V, with overnight stops at the end of the first day at one of the locations Q or R, and at the end of the second day at S, T or U. The network shows the journey times, in hours, for each day of the journey.



The optimal route, known as the minimax route, is that in which the longest day's journey is as small as possible.

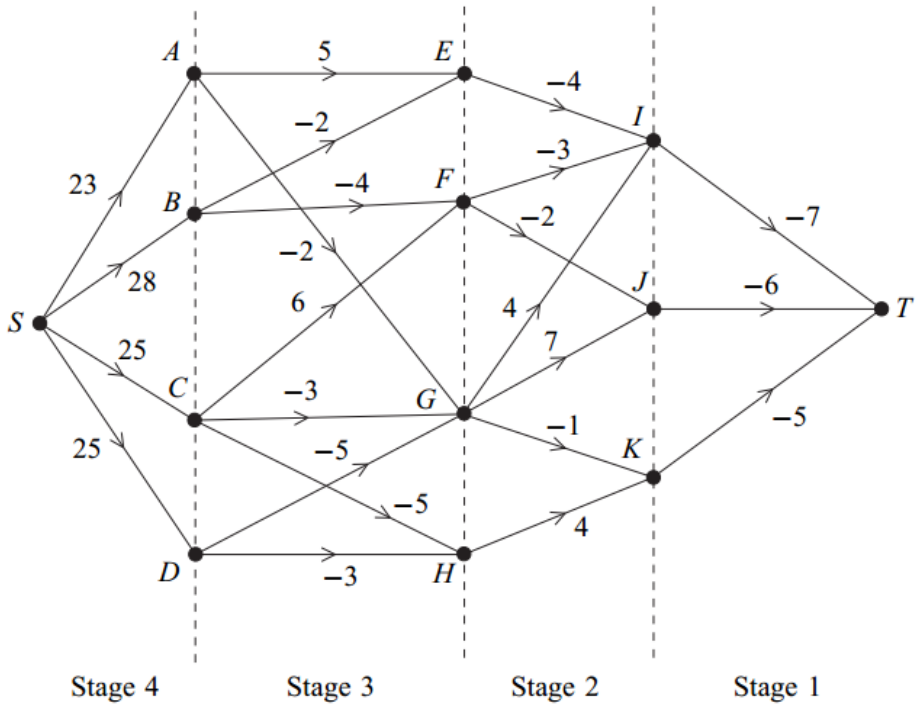
(a) Explain why the route $PQSV$ is better than the route $PQTV$. *(2 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from V**, to find the optimal (minimax) route from P to V.

You should indicate the calculations as well as the values at stages 2 and 3. *(8 marks)*

January 2011

5 Each path from S to T in the network below represents a possible way of using the internet to buy a ticket for a particular event. The number on each edge represents a charge, in pounds, with a negative value representing a discount. For example, the path $SAEIT$ represents a ticket costing $23 + 5 - 4 - 7 = 17$ pounds.



(a) **By working backwards from T** and completing the table on **Figure 4**, use dynamic programming to find the minimum weight of all paths from S to T. *(6 marks)*

(b) State the minimum cost of a ticket for the event and the paths corresponding to this minimum cost. *(3 marks)*

Stage	State	From	Value	
1	I	T	-7	
	J	T	-6	
	K	T	-5	
2	E	I	$-7-4=-11$	
	F	I		
		J		
	G	I		
		J		
		K		
	H	K		
3				

June 2011

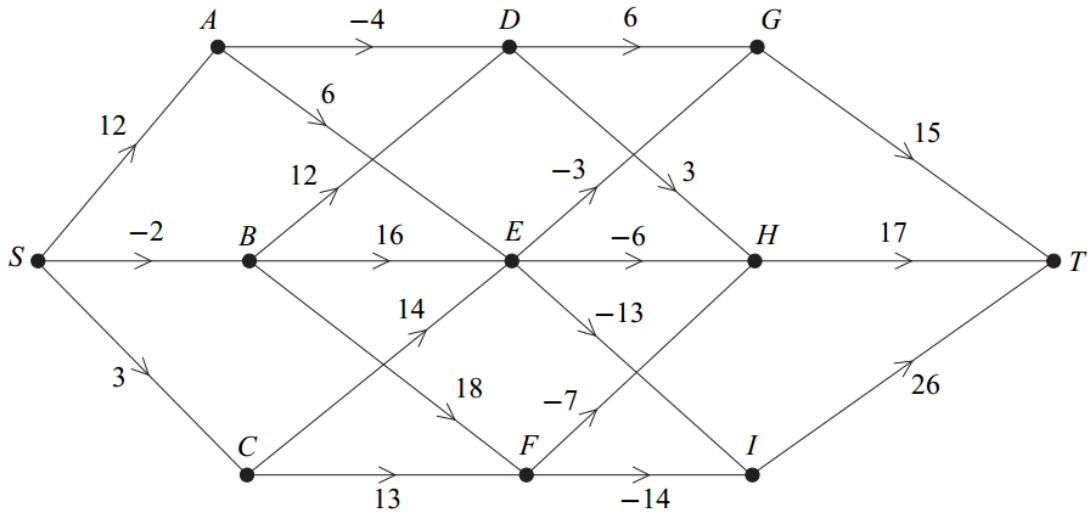
6 Bob is planning to build four garden sheds, A, B, C and D, at the rate of one per day. The order in which they are built is a matter of choice, but the costs will vary because some of the materials left over from making one shed can be used for the next one. The expected profits, in pounds, are given in the table below.

Day	Already built	Expected profit (£)			
		A	***B***	***C***	***D***
Monday	–	50	65	70	80
Tuesday	***A***	–	72	83	84
	B	60	–	80	83
	C	57	68	–	85
	D	62	70	81	–
Wednesday	***A* and *B***	–	–	84	88
	A* and *C	–	71	–	82
	A* and *D	–	74	83	–
	B* and *C	65	–	–	86
	B* and *D	69	–	85	–
	C* and *D	66	73	–	–
Thursday	***A*, *B* and *C***	–	–	–	90
	A*, *B* and *D	–	–	87	–
	A*, *C* and *D	–	76	–	–
	B*, *C* and *D	70	–	–	–

By completing the table of values opposite, or otherwise, use dynamic programming, **working backwards from Thursday**, to find the building schedule that maximises the total expected profit. *(9 marks)*

January 2012

5 A firm is considering various strategies for development over the next few years. In the network, the number on each edge is the expected profit, in millions of pounds, moving from one year to the next. A negative number indicates a loss because of building costs or other expenses. Each path from S to T represents a complete strategy.



(a) By completing the table on the page opposite, or otherwise, use dynamic programming **working backwards from *T*** to find the maximum weight of all paths from S to T. *(6 marks)*

(b) State the overall maximum profit and the paths from S to T corresponding to this maximum profit. *(3 marks)*

Stage	State	From	Calculation	Value
1	*G*	*T*		
	H	*T*		
	I	*T*		
2	*D*	*G*		
		H		
	E	*G*		
		H		
		I		
	F	*H*		
		I		
3				

Maximum profit is £..... million

Corresponding paths from *S* to *T* .....

June 2012

5 Dave plans to renovate three houses, A, B and C, at the rate of one per year. The order in which they are renovated is a matter of choice, but some costs vary over the three years. The expected costs, in thousands of pounds, are given in the table below.

Year	Already renovated	Cost		
		A	***B***	***C***
1	–	60	70	65
2	***A***	–	75	70
	B	55	–	60
	C	65	80	–
3	***A* and *B***	–	–	75
	A* and *C	–	80	–
	B* and *C	60	–	–

For tax reasons, Dave needs to choose the order for renovation so that the least annual cost is as large as possible. Solving the maximin problem will produce this optimum order for renovation.

(a) (i) State the least annual cost when the order of renovation is BAC.

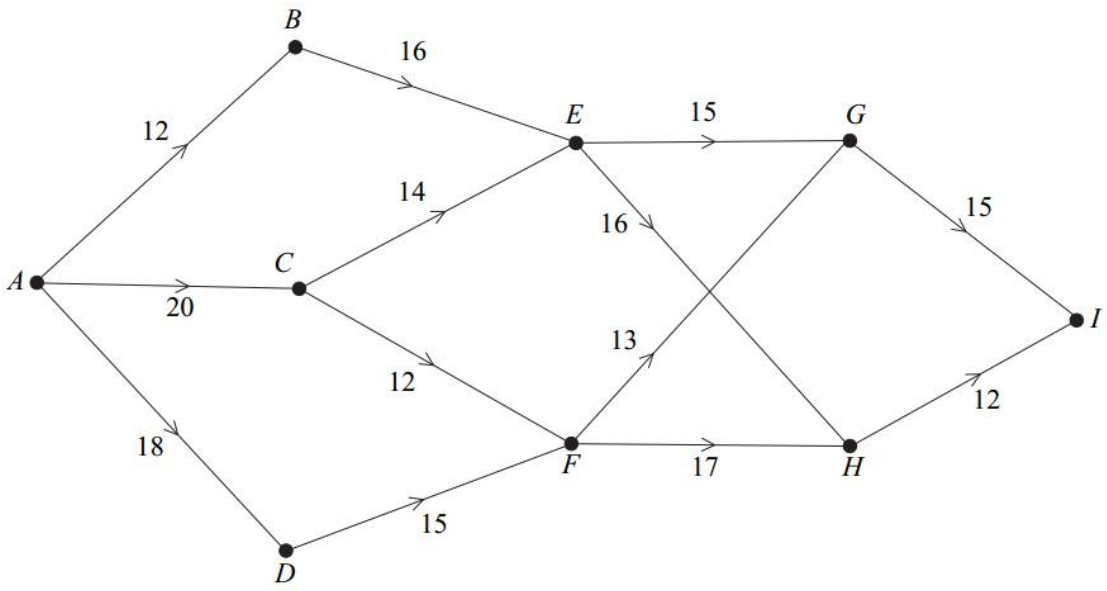
(ii) Determine, with a reason, whether the order ABC is better than the order BAC. *(3 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from Year 3**, to find the optimum order for renovation. *(7 marks)*

January 2013

7 The network below shows a system of one-way roads. The number on each edge represents the number of bags for recycling that can be collected by driving along that road.

A collector is to drive from A to I.



(a) **Working backwards from *I***, use dynamic programming to find the maximum number of bags that can be collected when driving from A to I.

You must complete the table opposite as your solution. *(7 marks)*

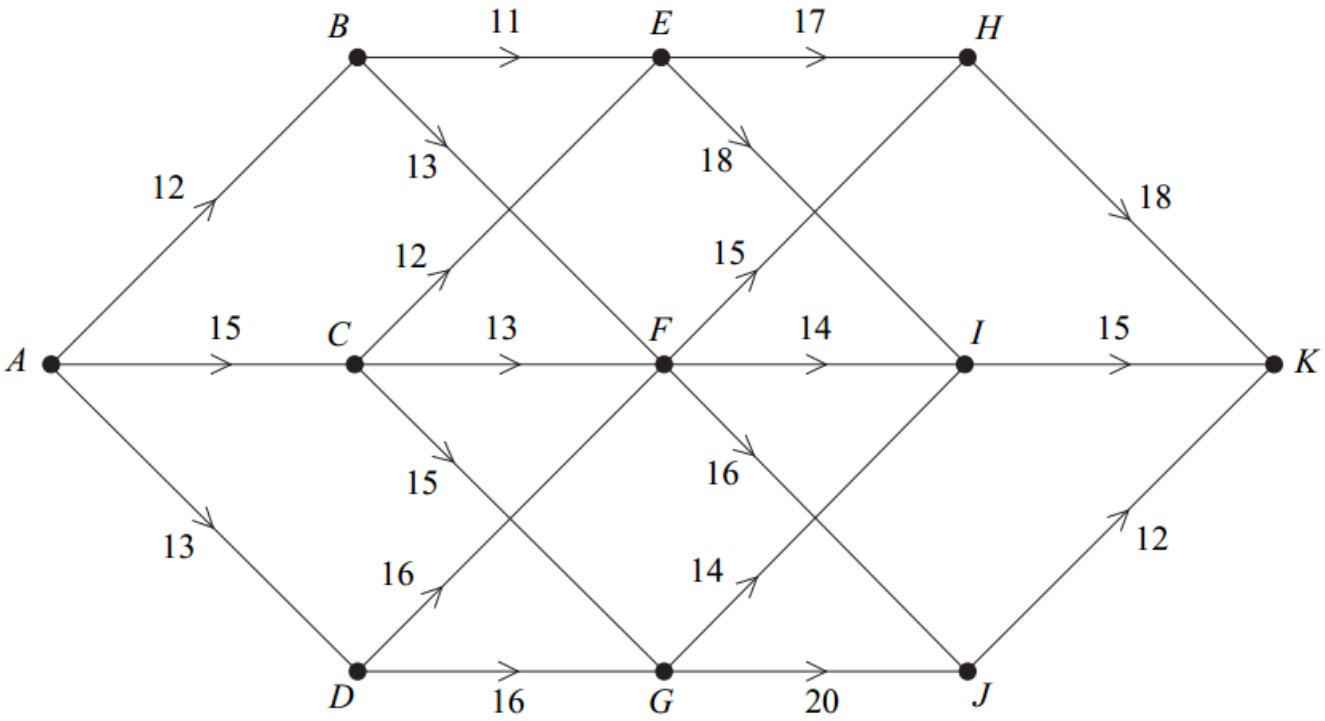
(b) State the route that the collector should take in order to collect the maximum number of bags. *(1 mark)*

Stage	State	From	Value
1	*G*	*I*	
	H	*I*	
2			

4 A haulage company, based in town A, is to deliver a tall statue to town K. The statue is being delivered on the back of a lorry.

The network below shows a system of roads. The number on each edge represents the height, in feet, of the lowest bridge on that road.

The company wants to ensure that the height of the lowest bridge along the route from A to K is maximised.



Working backwards from *K*, use dynamic programming to find the optimal route when driving from A to K.

You must complete the table opposite as your solution. *(9 marks)*

Stage	State	From	Value
1	*H*	*K*	
	I	*K*	
	J	*K*	
2			

Optimal route is .....