FP1: Complex Numbers

Past Exam Questions 2006 - 2013

Name:

January 2006

5 (a) (i) Calculate $(2 + i\sqrt{5})(\sqrt{5} - i)$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - i$ is a root of the equation

$$(2 + i\sqrt{5})\,z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

June 2006

6 It is given that $z = x + iy$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

$$(z + i)^*$$

where $(z + i)^*$ denotes the complex conjugate of $(z + i)$. *(2 marks)*

(b) Solve the equation

$$(z + i)^* = 2iz + 1$$

giving your answer in the form $a + bi$. *(5 marks)*

January 2007

1 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + i)^3$ in the form $a + bi$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + i$ satisfies the equation

$$x^3 + 2x - 4i = 0$$ *(2 marks)*

June 2007

3 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3iz^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3iz^* = 16$$ *(3 marks)*

January 2008

1 It is given that $z_1 = 2 + i$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3iy = z_1 + 4iz_1{}^*$$ *(4 marks)*

June 2008

2 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3iz + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3iz + 2z^* = 7 + 8i$$ *(3 marks)*

January 2009

2 The complex number $2 + 3\text{i}$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

June 2009

3 The complex number z is defined by

$$z = x + 2\text{i}$$

where x is real.

(a) Find, in terms of x, the real and imaginary parts of:

(i) z^2; *(3 marks)*

(ii) $z^2 + 2z^*$. *(2 marks)*

(b) Show that there is exactly one value of x for which $z^2 + 2z^*$ is real. *(2 marks)*

January 2010

2 The complex number z is defined by

$$z = 1 + \text{i}$$

(a) Find the value of z^2, giving your answer in its simplest form. *(2 marks)*

(b) Hence show that $z^8 = 16$. *(2 marks)*

(c) Show that $(z^*)^2 = -z^2$. *(2 marks)*

June 2010

2 It is given that $z = x + \text{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(1 - 2\text{i})z - z^*$$ *(4 marks)*

(b) Hence find the complex number z such that

$$(1 - 2\text{i})z - z^* = 10(2 + \text{i})$$ *(2 marks)*

January 2011

5 (a)		It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.	
	(i)	Calculate the value of ${z_1}^2$, giving your answer in the form $a + b\mathrm{i}$.	*(2 marks)*
	(ii)	Hence verify that z_1 is a root of the equation $$z^2 + z^* + \frac{1}{4} = 0$$	*(2 marks)*
(b)		Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**.	*(2 marks)*
(c)		Show that the equation in part **(a)(ii)** has two equal **real** roots.	*(2 marks)*

June 2011

3	It is given that $z = x + \mathrm{i}y$, where x and y are real.	
(a)	Find, in terms of x and y, the real and imaginary parts of $$(z - \mathrm{i})(z^* - \mathrm{i})$$	*(3 marks)*
(b)	Given that $$(z - \mathrm{i})(z^* - \mathrm{i}) = 24 - 8\mathrm{i}$$ find the two possible values of z.	*(4 marks)*

January 2012

3 (a)		Solve the following equations, giving each root in the form $a + b\mathrm{i}$:	
	(i)	$x^2 + 9 = 0$;	*(1 mark)*
	(ii)	$(x + 2)^2 + 9 = 0$.	*(1 mark)*
(b)	**(i)**	Expand $(1 + x)^3$.	*(1 mark)*
	(ii)	Express $(1 + 2\mathrm{i})^3$ in the form $a + b\mathrm{i}$.	*(3 marks)*
	(iii)	Given that $z = 1 + 2\mathrm{i}$, find the value of $$z^* - z^3$$	*(2 marks)*

June 2012

3	It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.		
(a)	Find, in terms of x and y, the real and imaginary parts of		
	$$\mathrm{i}(z+7) + 3(z^* - \mathrm{i})$$	*(3 marks)*	
(b)	Hence find the complex number z such that		
	$$\mathrm{i}(z+7) + 3(z^* - \mathrm{i}) = 0$$	*(3 marks)*	

January 2013

2 (a)	Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\mathrm{i}$, where p and q are integers.	*(3 marks)*
(b)	It is given that $z = \mathrm{i}(1+\mathrm{i})(2+\mathrm{i})$.	
(i)	Express z in the form $a + b\mathrm{i}$, where a and b are integers.	*(3 marks)*
(ii)	Find integers m and n such that $z + mz^* = n\mathrm{i}$.	*(3 marks)*

June 2013

4 (a)	It is given that $z = x + y\mathrm{i}$, where x and y are real numbers.	
(i)	Write down, in terms of x and y, an expression for $(z - 2\mathrm{i})^*$.	*(1 mark)*
(ii)	Solve the equation $$(z - 2\mathrm{i})^* = 4\mathrm{i}z + 3$$ giving your answer in the form $a + b\mathrm{i}$.	*(5 marks)*
(b)	It is given that $p + q\mathrm{i}$, where p and q are real numbers, is a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$.	
	Without finding the values of p and q, **state** why $p - q\mathrm{i}$ is **not** a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$.	*(1 mark)*