Core 2: Calculus

Past Paper Questions
2006 - 2013

Name:

Binomial Series

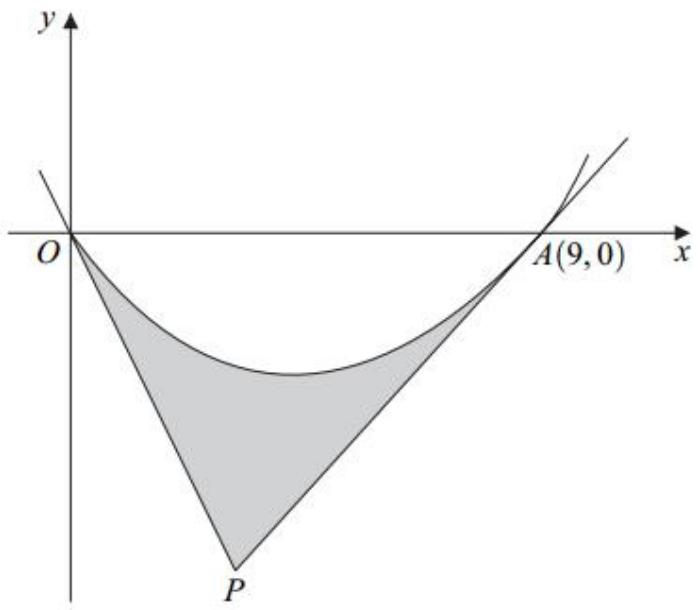
$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

January 2006

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$. *(5 marks)*

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

June 2006

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

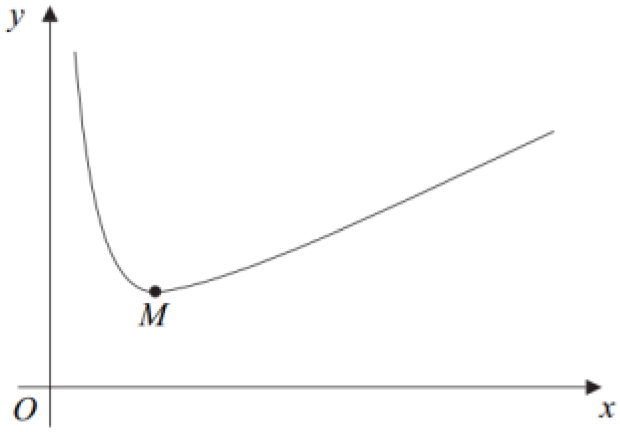
(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int \left(3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\right) dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

January 2007

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

June 2007

1 (i) Find $\int 3x^{\frac{1}{2}} \, dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}} \, dx$. *(2 marks)*

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

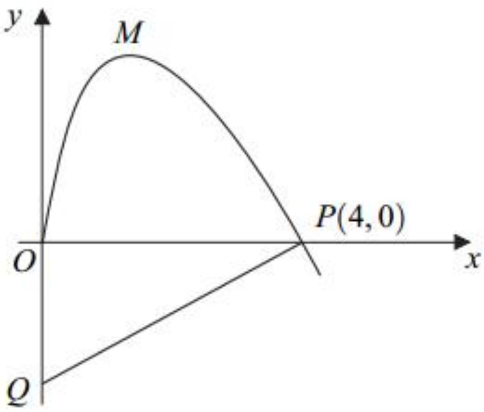
(c) Find $\frac{dy}{dx}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\displaystyle\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

June 2008

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

7 (a) The expression $\left(1 + \dfrac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

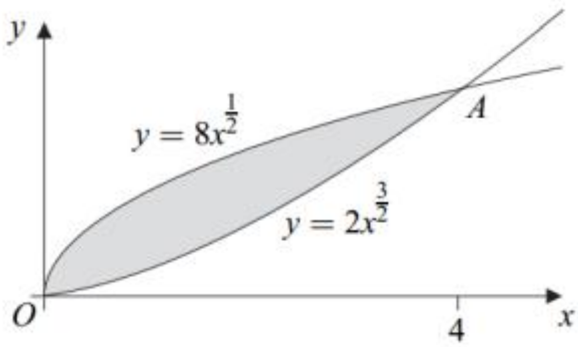
By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) (i) Hence find $\displaystyle\int \left(1 + \frac{4}{x^2}\right)^3 dx$. *(4 marks)*

(ii) Hence find the value of $\displaystyle\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 dx$. *(2 marks)*

January 2009

4 The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

(a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 4$. *(2 marks)*

(ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A. *(4 marks)*

(b) (i) Find $\int 8x^{\frac{1}{2}}\, dx$. *(2 marks)*

(ii) Find the area of the shaded region bounded by the two curves. *(4 marks)*

June 2009

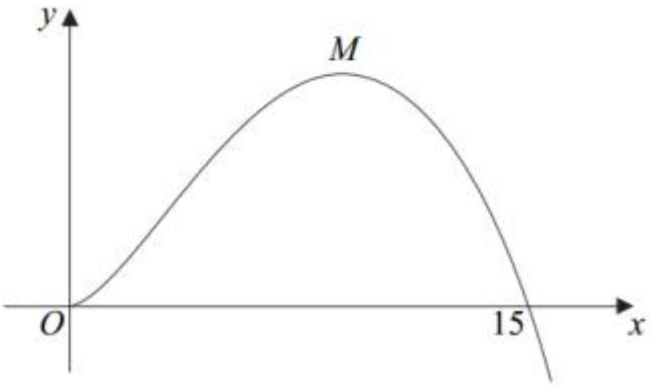
2 (a) Write down the value of n given that $\frac{1}{x^4} = x^n$. *(1 mark)*

(b) Expand $\left(1 + \frac{3}{x^2}\right)^2$. *(2 marks)*

(c) Hence find $\int \left(1 + \frac{3}{x^2}\right)^2 dx$. *(3 marks)*

(d) Hence find the exact value of $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 dx$. *(2 marks)*

5 The diagram shows part of a curve with a maximum point M.



The equation of the curve is

$$y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Hence find the coordinates of the maximum point M. *(4 marks)*

(c) The point $P(1, 14)$ lies on the curve. Show that the equation of the tangent to the curve at P is $y = 20x - 6$. *(3 marks)*

(d) The tangents to the curve at the points P and M intersect at the point R. Find the length of RM. *(3 marks)*

January 2010

2 At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx$. *(3 marks)*

(c) Hence find the equation of the curve, given that the curve passes through the point $(1, 3)$. *(3 marks)*

5 A curve has equation $y = \frac{1}{x^3} + 48x$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the equation of each of the two tangents to the curve that are parallel to the x-axis. *(4 marks)*

(c) Find an equation of the normal to the curve at the point $(1, 49)$. *(3 marks)*

June 2010

4 (a) The expression $\left(1 - \frac{1}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} - \frac{1}{x^6}$$

Find the values of the integers p and q. *(2 marks)*

(b) (i) Hence find $\int \left(1 - \frac{1}{x^2}\right)^3 dx$. *(4 marks)*

(ii) Hence find the value of $\int_{\frac{1}{2}}^{1} \left(1 - \frac{1}{x^2}\right)^3 dx$. *(2 marks)*

6 A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, \quad x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\frac{dy}{dx}$. *(2 marks)*

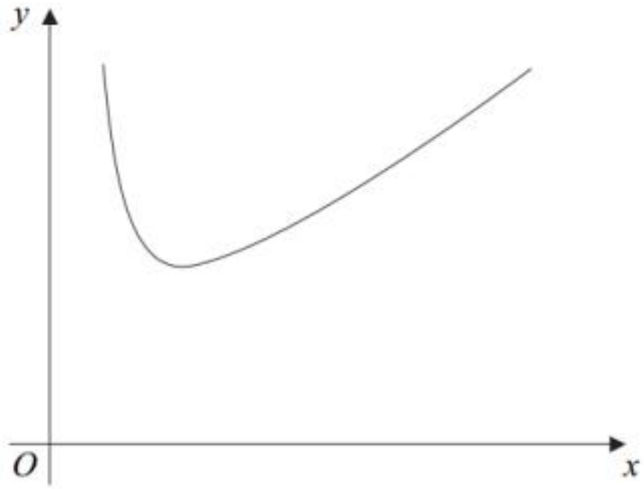
(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$. *(4 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(ii) Hence deduce that the curve C has no maximum points. *(2 marks)*

January 2011

7 A curve C is defined for $x > 0$ by the equation $y = x + 3 + \frac{8}{x^4}$ and is sketched below.



(a) Given that $y = x + 3 + \frac{8}{x^4}$, find $\frac{dy}{dx}$. *(3 marks)*

(b) Find an equation of the tangent at the point on the curve C where $x = 1$. *(3 marks)*

(c) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(d) (i) Find $\int \left(x + 3 + \frac{8}{x^4} \right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$. *(2 marks)*

June 2011

3 (a) The expression $(2 + x^2)^3$ can be written in the form

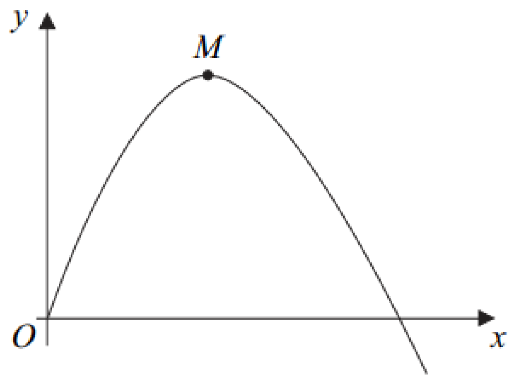
$$8 + px^2 + qx^4 + x^6$$

Show that $p = 12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\int \frac{(2 + x^2)^3}{x^4} dx$. *(5 marks)*

(ii) Hence find the exact value of $\int_1^2 \frac{(2 + x^2)^3}{x^4} dx$. *(2 marks)*

5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

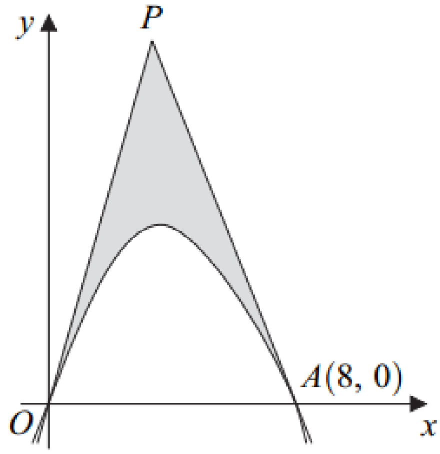
(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

June 2012

3 (a) Expand $\left(x^{\frac{3}{2}}-1\right)^2$. *(2 marks)*

(b) Hence find $\int\left(x^{\frac{3}{2}}-1\right)^2 \mathrm{d}x$. *(3 marks)*

(c) Hence find the value of $\int_1^4\left(x^{\frac{3}{2}}-1\right)^2 \mathrm{d}x$. *(2 marks)*

6 At the point (x, y), where $x>0$, the gradient of a curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-\frac{4}{x^2}-11$$

The point $P(2, 1)$ lies on the curve.

(a) (i) Verify that $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=2$. *(1 mark)*

(ii) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x=2$. *(4 marks)*

(iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. *(1 mark)*

(b) Find the equation of the curve. *(4 marks)*

January 2013

2 b) (i) Find $\int\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)\mathrm{d}x$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4\left(x^{-\frac{3}{2}}+6x^{\frac{1}{2}}\right)\mathrm{d}x$. *(2 marks)*

5 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \frac{8}{x^2}$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$. *(3 marks)*

(c) (i) Show that the stationary point M lies on the x-axis. *(3 marks)*

(ii) Hence **write down** the equation of the tangent to the curve at M. *(1 mark)*

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T. *(2 marks)*