FP1: Calculus

Past Exam Questions
2006 - 2013

Name:

8 (a) The function f is defined for all real values of x by

$$f(x) = x^3 + x^2 - 1$$

(i) Express $f(1+h) - f(1)$ in the form

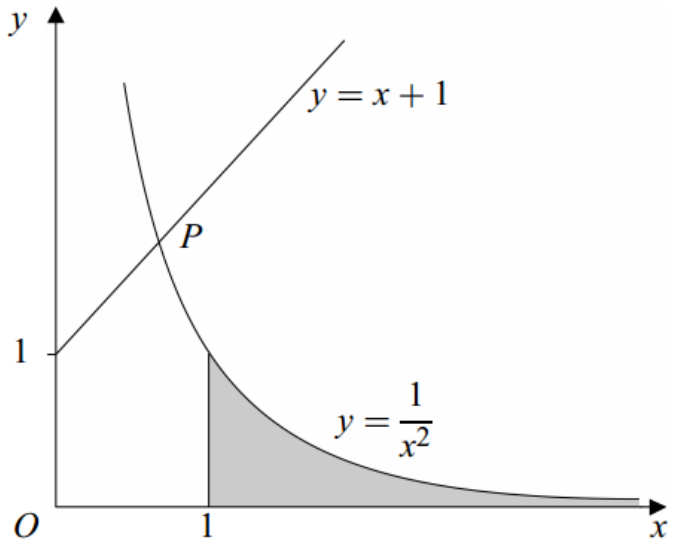
$$ph + qh^2 + rh^3$$

where p, q and r are integers. *(4 marks)*

(ii) Use your answer to part (a)(i) to find the value of $f'(1)$. *(2 marks)*

(b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$



The graphs intersect at the point P.

(i) Show that the x-coordinate of P satisfies the equation $f(x) = 0$, where f is the function defined in part (a). *(1 mark)*

(ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $f(x) = 0$, use the Newton–Raphson method to find a second approximation x_2 to the root. *(3 marks)*

(c) The region enclosed by the curve $y = \frac{1}{x^2}$, the line $x = 1$ and the x-axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. *(3 marks)*

January 2006

2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^9 \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(ii) $\int_0^9 \frac{1}{x\sqrt{x}}\,dx$. *(3 marks)*

(b) Explain briefly why the integrals in part (a) are improper integrals. *(1 mark)*

January 2007

7 The function f is defined for all real numbers by

$$f(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $f(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$g(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $g(x)$ gives a good approximation to $f(x)$ for small values of x.

(i) Show that $g(0.05)$ and $f(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = g(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $g'(0)$. *(1 mark)*

June 2007

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_0^1 (x^{\frac{1}{3}} + x^{-\frac{1}{3}})\,dx$; *(4 marks)*

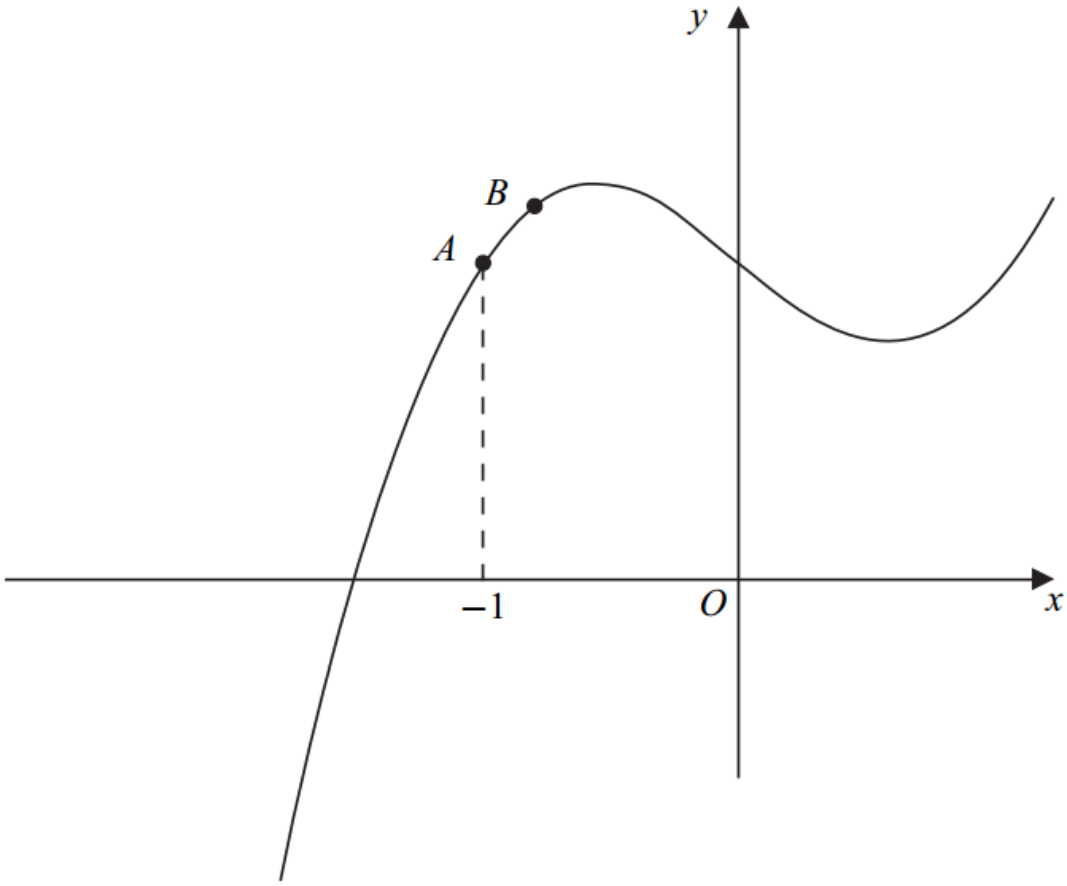
(b) $\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x}\,dx$. *(4 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$ *(3 marks)*

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

June 2008

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_9^\infty \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(b) $\int_9^\infty \frac{1}{x\sqrt{x}}\,dx$. *(4 marks)*

January 2009

8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\int_1^\infty x^{-\frac{3}{4}}\,dx$; *(3 marks)*

(b) $\int_1^\infty x^{-\frac{5}{4}}\,dx$; *(3 marks)*

(c) $\int_1^\infty (x^{-\frac{3}{4}} - x^{-\frac{5}{4}})\,dx$. *(1 mark)*

June 2009

2 A curve has equation

$$y = x^2 - 6x + 5$$

The points A and B on the curve have x-coordinates 2 and $2 + h$ respectively.

(a) Find, in terms of h, the gradient of the line AB, giving your answer in its simplest form. *(5 marks)*

(b) Explain how the result of part **(a)** can be used to find the gradient of the curve at A. State the value of this gradient. *(3 marks)*

January 2010

5 (a) Explain why $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$ is an improper integral. *(1 mark)*

(b) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$; *(3 marks)*

(ii) $\int_0^{\frac{1}{16}} x^{-\frac{5}{4}}\,\mathrm{d}x$. *(3 marks)*

June 2010

5 A curve has equation $y = x^3 - 12x$.

The point A on the curve has coordinates $(2, -16)$.

The point B on the curve has x-coordinate $2 + h$.

(a) Show that the gradient of the line AB is $6h + h^2$. *(4 marks)*

(b) Explain how the result of part **(a)** can be used to show that A is a stationary point on the curve. *(2 marks)*

January 2011

2 (a) Find, in terms of p and q, the value of the integral $\int_p^q \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

(b) Show that only one of the following improper integrals has a finite value, and find that value:

(i) $\int_0^2 \frac{2}{x^3}\,\mathrm{d}x$;

(ii) $\int_2^\infty \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

June 2011

6 (a) Expand $(5+h)^3$. *(1 mark)*

(b) A curve has equation $y = x^3 - x^2$.

(i) Find the gradient of the line passing through the point (5, 100) and the point on the curve for which $x = 5 + h$. Give your answer in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(4 marks)*

(ii) Show how the answer to part **(b)(i)** can be used to find the gradient of the curve at the point (5, 100). State the value of this gradient. *(2 marks)*

January 2012

2 Show that only one of the following improper integrals has a finite value, and find that value:

(a) $\int_8^\infty x^{-\frac{2}{3}}\,dx$;

(b) $\int_8^\infty x^{-\frac{4}{3}}\,dx$. *(5 marks)*

June 2012

2 A curve has equation $y = x^4 + x$.

(a) Find the gradient of the line passing through the point $(-2, 14)$ and the point on the curve for which $x = -2 + h$. Give your answer in the form

$$p + qh + rh^2 + h^3$$

where p, q and r are integers. *(5 marks)*

(b) Show how the answer to part **(a)** can be used to find the gradient of the curve at the point $(-2, 14)$. State the value of this gradient. *(2 marks)*

January 2013

4 Show that the improper integral $\int_{25}^\infty \frac{1}{x\sqrt{x}}\,dx$ has a finite value and find that value. *(4 marks)*

5 (a) A curve has equation $y = 2x^2 - 5x$.

The point P on the curve has coordinates $(1, -3)$.

The point Q on the curve has x-coordinate $1 + h$.

(i) Show that the gradient of the line PQ is $2h - 1$. *(3 marks)*

(ii) Explain how the result of part **(a)(i)** can be used to show that the tangent to the curve at the point P is parallel to the line $x + y = 0$. *(2 marks)*

(b) For the improper integral $\int_1^\infty x^{-4}(2x^2 - 5x)\,dx$, either show that the integral has a finite value and state its value, or explain why the integral does not have a finite value. *(3 marks)*