# Core 2: Binomial Expansion 

## Past Paper Questions 2006-2012

Name:

Binomial Series

$$
\begin{aligned}
& (a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n} \quad(n \in \mathbb{N}) \\
& \text { where }\binom{n}{r}={ }^{n} \mathrm{C}_{r}=\frac{n!}{r!(n-r)!}
\end{aligned}
$$

4 (a) The expression $(1-2 x)^{4}$ can be written in the form

$$
1+p x+q x^{2}-32 x^{3}+16 x^{4}
$$

By using the binomial expansion, or otherwise, find the values of the integers $p$ and $q$.
(3 marks)
(b) Find the coefficient of $x$ in the expansion of $(2+x)^{9}$.
(c) Find the coefficient of $x$ in the expansion of $(1-2 x)^{4}(2+x)^{9}$.

January 2007
7 (a) The first four terms of the binomial expansion of $(1+2 x)^{8}$ in ascending powers of $x$ are $1+a x+b x^{2}+c x^{3}$. Find the values of the integers $a, b$ and $c$.
(b) Hence find the coefficient of $x^{3}$ in the expansion of $\left(1+\frac{1}{2} x\right)(1+2 x)^{8}$.

6 (a) Using the binomial expansion, or otherwise:
(i) express $(1+x)^{3}$ in ascending powers of $x$;
(ii) express $(1+x)^{4}$ in ascending powers of $x$.
(b) Hence, or otherwise:
(i) express $(1+4 x)^{3}$ in ascending powers of $x$;
(ii) express $(1+3 x)^{4}$ in ascending powers of $x$.
(c) Show that the expansion of

$$
(1+3 x)^{4}-(1+4 x)^{3}
$$

can be written in the form

$$
p x^{2}+q x^{3}+r x^{4}
$$

where $p, q$ and $r$ are integers.

7 (a) The expression $\left(1+\frac{4}{x^{2}}\right)^{3}$ can be written in the form

$$
1+\frac{p}{x^{2}}+\frac{q}{x^{4}}+\frac{64}{x^{6}}
$$

By using the binomial expansion, or otherwise, find the values of the integers $p$ and $q$.

January 2009
5 (a) By using the binomial expansion, or otherwise, express $(1+2 x)^{4}$ in the form

$$
1+a x+b x^{2}+c x^{3}+16 x^{4}
$$

where $a, b$ and $c$ are integers.
(b) Hence show that $(1+2 x)^{4}+(1-2 x)^{4}=2+48 x^{2}+32 x^{4}$.

January 2010
7 (a) The first four terms of the binomial expansion of $(1+2 x)^{7}$ in ascending powers of $x$ are $1+a x+b x^{2}+c x^{3}$. Find the values of the integers $a, b$ and $c$.
(b) Hence find the coefficient of $x^{3}$ in the expansion of $\left(1-\frac{1}{2} x\right)^{2}(1+2 x)^{7}$.

4 (a) The expression $\left(1-\frac{1}{x^{2}}\right)^{3}$ can be written in the form

$$
1+\frac{p}{x^{2}}+\frac{q}{x^{4}}-\frac{1}{x^{6}}
$$

Find the values of the integers $p$ and $q$.

5 (a) Using the binomial expansion, or otherwise, express $(1-x)^{3}$ in ascending powers of $x$.
(2 marks)
(b) Show that the expansion of

$$
(1+y)^{4}-(1-y)^{3}
$$

is

$$
7 y+p y^{2}+q y^{3}+y^{4}
$$

where $p$ and $q$ are constants to be found.

June 2011
3 (a) The expression $\left(2+x^{2}\right)^{3}$ can be written in the form

$$
8+p x^{2}+q x^{4}+x^{6}
$$

Show that $p=12$ and find the value of the integer $q$.

January 2012
5 The first four terms in the binomial expansion of $\left(1+\frac{x}{3}\right)^{6}$ are $1+a x+b x^{2}+c x^{3}$.
Find the values of the constants $a, b$ and $c$, giving your answers in their simplest form.
(4 marks)

January 2013
8 (a) Expand $\left(1+\frac{4}{x}\right)^{2}$.
(1 mark)
(b) The first four terms of the binomial expansion of $\left(1+\frac{x}{4}\right)^{8}$ in ascending powers of $x$ are $1+a x+b x^{2}+c x^{3}$. Find the values of the constants $a, b$ and $c$.
(c) Hence find the coefficient of $x$ in the expansion of $\left(1+\frac{4}{x}\right)^{2}\left(1+\frac{x}{4}\right)^{8} . \quad$ (4 marks)

