Core 2: Binomial Expansion

Past Paper Questions 2006 - 2012

Name:

Binomial Series

$$(a+b)^{n} = a^{n} + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^{2} + \dots + \binom{n}{r} a^{n-r} b^{r} + \dots + b^{n} \qquad (n \in \mathbb{N})$$
where $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

4 (a) The expression $(1-2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

- (b) Find the coefficient of x in the expansion of $(2+x)^9$. (2 marks)
- (c) Find the coefficient of x in the expansion of $(1 2x)^4 (2 + x)^9$. (3 marks)

January 2007

- 7 (a) The first four terms of the binomial expansion of $(1+2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. (4 marks)
 - (b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. (3 marks)

January 2008

6 (a) Using the binomial expansion, or otherwise:

- (i) express $(1+x)^3$ in ascending powers of x; (2 marks)
- (ii) express $(1+x)^4$ in ascending powers of x. (2 marks)
- (b) Hence, or otherwise:
 - (i) express $(1+4x)^3$ in ascending powers of x; (2 marks)
 - (ii) express $(1+3x)^4$ in ascending powers of x. (2 marks)
- (c) Show that the expansion of

$$(1+3x)^4 - (1+4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p, q and r are integers.

(2 marks)

7 (a) The expression $\left(1 + \frac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

January 2009

5 (a) By using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form

$$1 + ax + bx^2 + cx^3 + 16x^4$$

where a, b and c are integers.

(4 marks)

(b) Hence show that $(1+2x)^4 + (1-2x)^4 = 2 + 48x^2 + 32x^4$. (3 marks)

January 2010

- 7 (a) The first four terms of the binomial expansion of $(1+2x)^7$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the integers a, b and c. (4 marks)
 - (b) Hence find the coefficient of x^3 in the expansion of $\left(1 \frac{1}{2}x\right)^2 (1 + 2x)^7$. (4 marks)

June 2010

4 (a) The expression $\left(1 - \frac{1}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} - \frac{1}{x^6}$$

Find the values of the integers p and q.

(2 marks)

- Using the binomial expansion, or otherwise, express $(1-x)^3$ in ascending powers of x.
 - **(b)** Show that the expansion of

$$(1+y)^4 - (1-y)^3$$

is

$$7y + py^2 + qy^3 + y^4$$

where p and q are constants to be found.

(4 marks)

June 2011

3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8 + px^2 + qx^4 + x^6$$

Show that p = 12 and find the value of the integer q.

(3 marks)

January 2012

The first four terms in the binomial expansion of $\left(1+\frac{x}{3}\right)^6$ are $1+ax+bx^2+cx^3$. Find the values of the constants a, b and c, giving your answers in their simplest form. (4 marks)

January 2013

- 8 (a) Expand $\left(1 + \frac{4}{x}\right)^2$. (1 mark)
 - (b) The first four terms of the binomial expansion of $\left(1 + \frac{x}{4}\right)^{\delta}$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the constants a, b and c. (4 marks)
 - (c) Hence find the coefficient of x in the expansion of $\left(1 + \frac{4}{x}\right)^2 \left(1 + \frac{x}{4}\right)^8$. (4 marks)