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# FP1: Asymptotes

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Past Exam Questions  
2006 - 2013

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January 2006

**4** A curve has equation

$$y = \frac{6x}{x-1}$$

- (a) Write down the equations of the two asymptotes to the curve. *(2 marks)*
- (b) Sketch the curve and the two asymptotes. *(4 marks)*
- (c) Solve the inequality

$$\frac{6x}{x-1} < 3 \quad (4 \text{ marks})$$

June 2006

**9** A curve  $C$  has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

- (a) (i) Write down the coordinates of the points where  $C$  intersects the  $x$ -axis. *(2 marks)*
- (ii) Write down the equations of all the asymptotes of  $C$ . *(3 marks)*
- (b) (i) Show that, if the line  $y = k$  intersects  $C$ , then

$$(k-1)(k-4) \geq 0 \quad (5 \text{ marks})$$

- (ii) Given that there is only one stationary point on  $C$ , find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(3 marks)*

- (c) Sketch the curve  $C$ . *(3 marks)*

January 2007

**5** A curve has equation

$$y = \frac{x}{x^2-1}$$

- (a) Write down the equations of the three asymptotes to the curve. *(3 marks)*
- (b) Sketch the curve.
- (You are given that the curve has no stationary points.) *(4 marks)*

- (c) Solve the inequality

$$\frac{x}{x^2-1} > 0 \quad (3 \text{ marks})$$

June 2007

7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

- (a) Write down the equations of the two asymptotes to the curve. (2 marks)
- (b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. (5 marks)
- (c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3 \quad (2 \text{ marks})$$

January 2008

9 A curve  $C$  has equation

$$y = \frac{2}{x(x - 4)}$$

- (a) Write down the equations of the three asymptotes of  $C$ . (3 marks)
- (b) The curve  $C$  has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.  
(No credit will be given for solutions based on differentiation.) (6 marks)
- (c) Sketch the curve  $C$ . (3 marks)

June 2008

7 A curve  $C$  has equation

$$y = 7 + \frac{1}{x + 1}$$

- (a) Define the translation which transforms the curve with equation  $y = \frac{1}{x}$  onto the curve  $C$ . (2 marks)
- (b) (i) Write down the equations of the two asymptotes of  $C$ . (2 marks)
- (ii) Find the coordinates of the points where the curve  $C$  intersects the coordinate axes. (3 marks)
- (c) Sketch the curve  $C$  and its two asymptotes. (3 marks)

January 2009

**6** A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. (3 marks)

(ii) State the coordinates of the points at which the curve intersects the  $x$ -axis. (1 mark)

(iii) Sketch the curve.

(You are given that the curve has no stationary points.) (4 marks)

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0 \quad (2 \text{ marks})$$

June 2009

**8** A curve has equation

$$y = \frac{x^2}{(x-1)(x-5)}$$

(a) Write down the equations of the three asymptotes to the curve. (3 marks)

(b) Show that the curve has no point of intersection with the line  $y = -1$ . (3 marks)

(c) (i) Show that, if the curve intersects the line  $y = k$ , then the  $x$ -coordinates of the points of intersection must satisfy the equation

$$(k-1)x^2 - 6kx + 5k = 0 \quad (2 \text{ marks})$$

(ii) Show that, if this equation has equal roots, then

$$k(4k+5) = 0 \quad (2 \text{ marks})$$

(d) Hence find the coordinates of the two stationary points on the curve. (5 marks)

January 2010

**7** A curve  $C$  has equation  $y = \frac{1}{(x-2)^2}$ .

(a) (i) Write down the equations of the asymptotes of the curve  $C$ . (2 marks)

(ii) Sketch the curve  $C$ . (2 marks)

**7 (a) (i)** Write down the equations of the two asymptotes of the curve  $y = \frac{1}{x-3}$ . (2 marks)

**(ii)** Sketch the curve  $y = \frac{1}{x-3}$ , showing the coordinates of any points of intersection with the coordinate axes. (2 marks)

**(iii)** On the same axes, again showing the coordinates of any points of intersection with the coordinate axes, sketch the line  $y = 2x - 5$ . (1 mark)

**(b) (i)** Solve the equation

$$\frac{1}{x-3} = 2x - 5 \quad (3 \text{ marks})$$

**(ii)** Find the solution of the inequality

$$\frac{1}{x-3} < 2x - 5 \quad (2 \text{ marks})$$

**7** A graph has equation

$$y = \frac{x-4}{x^2+9}$$

**(a)** Explain why the graph has no vertical asymptote and give the equation of the horizontal asymptote. (2 marks)

**(b)** Show that, if the line  $y = k$  intersects the graph, the  $x$ -coordinates of the points of intersection of the line with the graph must satisfy the equation

$$kx^2 - x + (9k + 4) = 0 \quad (2 \text{ marks})$$

**(c)** Show that this equation has real roots if  $-\frac{1}{2} \leq k \leq \frac{1}{18}$ . (5 marks)

**(d)** Hence find the coordinates of the two stationary points on the curve.

(No credit will be given for methods involving differentiation.) (6 marks)

June 2011

**8** A curve has equation  $y = \frac{1}{x^2 - 4}$ .

**(a) (i)** Write down the equations of the three asymptotes of the curve. *(3 marks)*

**(ii)** Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes. *(4 marks)*

**(b)** Hence, or otherwise, solve the inequality

$$\frac{1}{x^2 - 4} < -2$$

*(3 marks)*

January 2012

**9** A curve has equation

$$y = \frac{x}{x - 1}$$

**(a)** Find the equations of the asymptotes of this curve. *(2 marks)*

**(b)** Given that the line  $y = -4x + c$  intersects the curve, show that the  $x$ -coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c + 3)x + c = 0$$

*(3 marks)*

**(c)** It is given that the line  $y = -4x + c$  is a tangent to the curve.

**(i)** Find the two possible values of  $c$ .

*(No credit will be given for methods involving differentiation.) (3 marks)*

**(ii)** For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. *(4 marks)*

**5** The curve  $C$  has equation  $y = \frac{x}{(x+1)(x-2)}$ .

The line  $L$  has equation  $y = -\frac{1}{2}$ .

**(a)** Write down the equations of the asymptotes of  $C$ . *(3 marks)*

**(b)** The line  $L$  intersects the curve  $C$  at two points. Find the  $x$ -coordinates of these two points. *(2 marks)*

**(c)** Sketch  $C$  and  $L$  on the same axes.  
(You are given that the curve  $C$  has no stationary points.) *(3 marks)*

**(d)** Solve the inequality

$$\frac{x}{(x+1)(x-2)} \leq -\frac{1}{2} \quad (3 \text{ marks})$$

**9** A curve has equation

$$y = \frac{x^2 - 2x + 1}{x^2 - 2x - 3}$$

**(a)** Find the equations of the three asymptotes of the curve. *(3 marks)*

**(b) (i)** Show that if the line  $y = k$  intersects the curve then

$$(k-1)x^2 - 2(k-1)x - (3k+1) = 0 \quad (1 \text{ mark})$$

**(ii)** Given that the equation  $(k-1)x^2 - 2(k-1)x - (3k+1) = 0$  has real roots, show that

$$k^2 - k \geq 0 \quad (3 \text{ marks})$$

**(iii)** Hence show that the curve has only one stationary point and find its coordinates.  
(No credit will be given for solutions based on differentiation.) *(4 marks)*

**(c)** Sketch the curve and its asymptotes. *(3 marks)*