Core 1 - Differentiaition

Challenge 1

The function f is defined for all real values of x by

$$f(x) = (x^2 + 4)(2x - 1) .$$

(a) Prove that the curve with equation $y = f(x)$ crosses the x-axis at only one point and state the x-coordinate of this point. *(2 marks)*

(b) (i) Differentiate $f(x)$ with respect to x to obtain $f'(x)$. *(4 marks)*

(ii) Hence show that the gradient of the curve $y = f(x)$ is 12 at the point where $x = 1$. *(2 marks)*

(iii) Prove that the curve $y = f(x)$ has no stationary point. *(2 marks)*

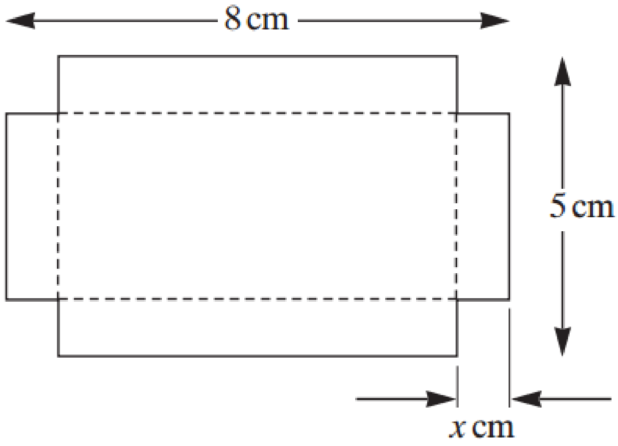
(c) The curve $y = f(x)$ intersects the line $y = x$ at only one point B.

(i) Show that the x-coordinate of B satisfies the equation

$$2x^3 - x^2 + 7x - 4 = 0 .$$ *(1 mark)*

Challenge 2

Small trays are to be made from rectangular pieces of card. Each piece of card is 8 cm by 5 cm and the tray is formed by removing squares of side x cm from each corner and folding the remaining card along the dashed lines, as shown in the diagram, to form an open-topped box.



(a) Explain why $0 < x < 2.5$. *(1 mark)*

(b) Show that the volume, V cm^3, of a tray is given by

$$V = 4x^3 - 26x^2 + 40x.$$ *(3 marks)*

(c) Find the value of x for which $\frac{dV}{dx} = 0$. *(5 marks)*

(d) Calculate the greatest possible volume of a tray. *(1 mark)*

Challenge 3

An office worker can leave home at any time between 6.00 am and 10.00 am each morning. When he leaves home x **hours** after 6.00 am ($0 \leqslant x \leqslant 4$), his journey time to the office is y **minutes**, where

$$y = x^4 - 8x^3 + 16x^2 + 8.$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(b) Find the **three** values of x for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$. *(4 marks)*

(c) Show that y has a maximum value when $x = 2$. *(3 marks)*

(d) Find the time at which the office worker arrives at the office on a day when his journey time is a maximum. *(2 marks)*

Final Challenge

The size of a population, P, of birds on an island is modelled by

$$P = 59 + 117t + 57t^2 - t^3,$$

where t is the time in years after 1970.

(a) Find $\frac{dP}{dt}$. *(2 marks)*

(b) (i) Find the positive value of t for which P has a stationary value. *(3 marks)*

(ii) Determine whether this stationary value is a maximum or a minimum. *(2 marks)*

(c) (i) State the year when the model predicts that the population will reach its maximum value. *(1 mark)*

(ii) Determine what the model predicts will happen in the year 2029. *(1 mark)*