Find the equation of the tangent to the curve $y = \dfrac{2 + x}{\cos x}$ at the point on the curve where $x = 0$.

(6 marks)

Q	Solution	Marks	Total	Comments
3	$y' = \dfrac{\cos x + (2+x)\sin x}{\cos^2 x}$	M1A1		Product rule acceptable $\dfrac{\cos x - (2+x)(-\sin x)}{\cos^2 x}$ M1A1 If simplified incorrectly M1A0
	$x = 0, \quad y' = 1$ $x = 0, \quad y = 2$	A1F B1		
	Tangent: $\dfrac{y-2}{x} = 1$ $y = 2 + x$	m1A1F	6	f.t. non-zero / non-infinite gradient m1 depends on first M1
	Total		**6**	

(a) Differentiate:

(i) $2x^{\frac{1}{2}}$;

(ii) $\ln(x + 1)$. *(3 marks)*

(b) Hence show that $\int_1^4 \left(x^{-\frac{1}{2}} + \frac{1}{x+1}\right) \mathrm{d}x = 2 + \ln \frac{5}{2}$. *(5 marks)*

3 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2x^{\frac{1}{2}}\right) = kx^{-\frac{1}{2}}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \frac{1}{x}$	M1		
(i)	$k = 1$	A1		Allow $2 \times \frac{1}{2}$
(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(x+1)) = \frac{1}{x+1}$	A1	3	
(b)	$\int\left(x^{-\frac{1}{2}} + \frac{1}{x+1}\right)\mathrm{d}x = 2x^{\frac{1}{2}} + \ln(x+1)$	M1		Allow M1 if at least one term correct
	Substituting $x = 4$ or $x = 1$	m1		in at least one correct term
	Both substitutions and subtraction	m1		ditto; condone subtraction wrong way round.
	Use of log law	m1		Accept $(4 + \ln 5) - (2 + \ln 2) = 2 + \ln\frac{5}{2}$
	Answer $2 + \ln\frac{5}{2}$	A1	5	convincingly found (AG)
	Total		**8**	

(a) By using the chain rule, or otherwise, find $\dfrac{dy}{dx}$ when $y = \ln(x^2 + 9)$. *(3 marks)*

(b) Hence show that $\displaystyle\int_0^3 \frac{x}{x^2 + 9}\,dx = \frac{1}{2}\ln 2$. *(3 marks)*

(c) Show that $\displaystyle\int_0^3 \frac{x + 1}{x^2 + 9}\,dx = \frac{1}{2}\ln 2 + \frac{\pi}{12}$. *(4 marks)*

Q	Solution	Marks	Total	Comments
4 (a)	$y = \ln(x^2 + 9)$			
	let $u = x^2 + 9$ then $\dfrac{du}{dx} = 2x$ and $y = \ln u$ $\therefore \dfrac{dy}{du} = \dfrac{1}{u} = \dfrac{1}{x^2 + 9}$	M1		
	$\therefore \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \dfrac{1}{x^2 + 9} \times 2x$	M1		Use of chain rule
	$= \dfrac{2x}{x^2 + 9}$	A1	3	CAO
(b)	$\displaystyle\int_0^3 \frac{x}{x^2 + 9}\,dx = \left[\frac{1}{2}\ln(x^2 + 9)\right]_0^3$	M1		
	$= \dfrac{1}{2}\ln 18 - \dfrac{1}{2}\ln 9$	A1		
	$= \dfrac{1}{2}\ln 2$	A1	3	AG
(c)	$\displaystyle\int_0^3 \frac{x + 1}{x^2 + 9}\,dx = \int_0^3 \frac{x}{x^2 + 9}\,dx + \int_0^3 \frac{1}{x^2 + 9}\,dx$	M1		Attempted
	$= \dfrac{1}{2}\ln 2 + \dfrac{1}{3}\left[\tan^{-1}\left(\dfrac{x}{3}\right)\right]_0^3$	A1		
	$= = \dfrac{1}{2}\ln 2 + \dfrac{1}{3}\left[\tan^{-1}(-1) - \tan^{-1}(0)\right]$	M1		Limits used in correct expression
	$= \dfrac{1}{2}\ln 2 + \dfrac{\pi}{12}$	A1	4	AG
	Total		**10**	

A curve has equation

$$y = e^{2x} - 4x\ .$$

(a) Show that the x-coordinate of the stationary point on the curve is $\frac{1}{2}\ln 2$. Find the corresponding y-coordinate in the form $a + b\ln 2$, where a and b are integers to be determined. *(6 marks)*

(b) Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point. *(3 marks)*

Q	Solution	Marks	Total	Comments
6 (a)	Derivative of e^{2x} is ke^{2x}	M1		
	$y' = 2e^{2x} - 4$	A1		
	Attempt to solve $y' = 0$	M1		at least as far as $e^{2x} = 2$ OE
	Use of ln as inverse of exp	m1		verification can earn M1m1A0
	$x = \frac{1}{2}\ln 2$	A1		convincingly found (AG)
	$y = 2 - 2\ln 2$	B1	6	
(b)	Differentiation of their y'	m1		dependent on first M1
	$y'' = 4e^{2x}$	A1F		f.t numerical error
	$y'' > 0$ at the SP, so minimum point	B1F	3	allow their y''; condone incorrect pos value of $4e^{2x}$ at SP; f.t $y'' < 0$ at SP