FP1 Complex numbers Challenge

Challenge 1

The complex number z is equal to $x + iy$, where x and y are real numbers.

(a) Given that z^* is the conjugate of z, expand $(1 - i)z^*$ in terms of x and y. *(2 marks)*

(b) Given that

$$2(z - 1) = (1 - i)z^*$$

find the value of the complex number z. *(4 marks)*

Challenge 2

(a) Show that $(3 - \mathrm{i})^2 = 8 - 6\mathrm{i}$. *(1 mark)*

(b) The quadratic equation

$$az^2 + bz + 10\mathrm{i} = 0,$$

where a and b are real, has a root $3 - \mathrm{i}$.

(i) Show that $a = 3$ and find the value of b. *(6 marks)*

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + \mathrm{i}q$. *(3 marks)*

Challenge 3

It is given that $z = x + iy$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for z^*, the complex conjugate of z.
(1 mark)

(b) Find, in terms of x and y, the real and imaginary parts of

$$2z - iz^*$$ *(2 marks)*

(c) Find the complex number z such that

$$2z - iz^* = 3i$$ *(3 marks)*

Final Challenge

(a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + \mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + \mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*