2 The complex number z is equal to $x + \mathrm{i}y$, where x and y are real numbers.

(a) Given that z^* is the conjugate of z, expand $(1 - \mathrm{i})z^*$ in terms of x and y. *(2 marks)*

(b) Given that

$$2(z - 1) = (1 - \mathrm{i})z^*$$

find the value of the complex number z. *(4 marks)*

2(a)	$z^* = x - \mathrm{i}y$	M1		
	$(1-\mathrm{i})z^* = x - \mathrm{i}y - \mathrm{i}x - y$	A1	2	oe; $\mathrm{i}^2 = -1$ must be used
(b)	Equating $2(x + \mathrm{i}y - 1)$ to above	M1		
	Equating R and I parts	m1		
	Solving sim equations	m1		
	$x = 3$, $y = -1$ (so $z = 3 - \mathrm{i}$)	A1	4	
	Total		**6**	

1 (a) Show that $(3 - \mathrm{i})^2 = 8 - 6\mathrm{i}$. *(1 mark)*

(b) The quadratic equation

$$az^2 + bz + 10\mathrm{i} = 0,$$

where a and b are real, has a root $3 - \mathrm{i}$.

(i) Show that $a = 3$ and find the value of b. *(6 marks)*

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + \mathrm{i}q$. *(3 marks)*

Q	Solution	Marks	Total	Comments
1(a)	$(3-\mathrm{i})^2 = 9 - 6\mathrm{i} + \mathrm{i}^2 = 8 - 6\mathrm{i}$	B1	1	
(b)(i)	$a(8-6\mathrm{i}) + b(3-\mathrm{i}) + 10\mathrm{i} = 0$	M1		Substituting $3 - \mathrm{i}$ into quadratic.
	Equating R & I parts	M1A1		
	$8a + 3b = 0$			
	$-6a - b + 10 = 0$			
	Attempt to solve	M1		
	$a = 3, \quad b = -8$	A1A1F	6	$a = 3$ is AG If $a = 3$ is assumed, allow M1A1 for b
(ii)	Sum of roots $= -\dfrac{b}{a}$	M1		If sum of roots is -8 give M0
	or product $= \dfrac{c}{a}$			
	$\beta = -\dfrac{1}{3} + \mathrm{i}$	A1A1F	3	A1 for $-\dfrac{1}{3}$, A1 for $+\mathrm{i}$
	Total		**10**	

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for z^*, the complex conjugate of z. *(1 mark)*

(b) Find, in terms of x and y, the real and imaginary parts of

$$2z - \mathrm{i}z^*$$ *(2 marks)*

(c) Find the complex number z such that

$$2z - \mathrm{i}z^* = 3\mathrm{i}$$ *(3 marks)*

3(a)	$z^* = x - \mathrm{i}y$	B1	1	
(b)	$\mathrm{R} = 2x - y$	B1		$\mathrm{i}^2 = -1$ must be used
	$\mathrm{I} = -x + 2y$	B1	2	Condone $\mathrm{I} = \mathrm{i}(x + 2y)$; Answers may appear in (c)
(c)	Equating R and/or I parts Attempt to solve sim equations $z = 1 + 2\mathrm{i}$	M1 m1 A1	3	Allow $x = 1$, $y = 2$
	Total		**6**	

1 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + \mathrm{i})^3$ in the form $a + bi$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + \mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*

Q	Solution	Marks	Total	Comments
1(a)(i)	Roots are $\pm 4\mathrm{i}$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1 \pm 4\mathrm{i}$	M1A1	2	M1 for correct method
(b)(i)	$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1 + \mathrm{i})^3 = 1 + 3\mathrm{i} - 3 - \mathrm{i} = -2 + 2\mathrm{i}$	M1A1	2	M1 if $\mathrm{i}^2 = -1$ used
(iii)	$(1 + \mathrm{i})^3 + 2(1 + \mathrm{i}) - 4\mathrm{i}$	M1		with attempt to evaluate
	$\ldots = (-2 + 2\mathrm{i}) + (2 - 2\mathrm{i}) = 0$	A1	2	convincingly shown (**AG**)
	Total		**10**	