Circles

A circle has equation $x^2 + y^2 + 2x - 6y = 0$.

(a) Find the radius of the circle, and the coordinates of its centre. *(4 marks)*

(b) Find the equation of the tangent to the circle at the point (2, 4). *(5 marks)*

Q	Solution	Marks	Total	Comments
6 (a)	$(x+1)^2 + (y-3)^2 = 10$	M1A1		Completing square; not necessarily in final form shown here
	Centre $(-1, 3)$	A1√		} implies previous M1A1
	Radius $\sqrt{10}$	A1√	4	}
(b)	Slope of line through (2, 4) and their centre $((-1, 3))$	M1		**Alternative** Find gradient of tangent by differentiating eg $x^2 + y^2 + 2x - 6y = 0$
	$= +\frac{1}{3}$	A1√		$2x + 2yy' + 2 - 6y' = 0$ M1-must have $2yy'$ A1-fully correct
	$\Rightarrow$ slope of tangent $= -3$	B1√		Subs for slope $(= -3)$ B1√
	Tangent $\frac{y-4}{x-2} = -3$	M1		Then as on LHS for M1,A1
	$y + 3x = 10$	A1	5	(Any form)
	Total		**(9)**	

A circle has the equation

$$(x - 3)^2 + (y - 4)^2 = 16.$$

The point A has coordinates $\left(\frac{3}{5}, \frac{4}{5}\right)$.

(a) Show that A lies on the circle. *(1 mark)*

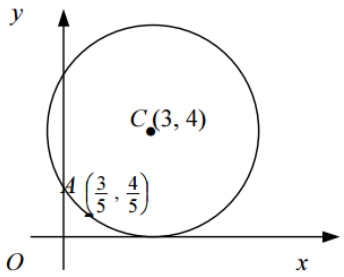
(b) Sketch the circle. *(2 marks)*

(c) Show that the normal to the circle at A passes through the origin. *(3 marks)*

(d) Find the equation of the tangent to the circle at A, giving your answer in the form

$$ax + by = c,$$

where a, b and c are integers. *(4 marks)*

3 (a)	$\left(\frac{3}{5}-3\right)^2+\left(\frac{4}{5}-4\right)^2=\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2=16$	B1	1	Or use of $x^2+y^2-6x-8y+9=0$
(b)		B1		Centre (PI)
		B1	2	Touching Ox
(c)	Gradient of OA, AC or OC found.	B1		This mark can be awarded in part (d) if not earned here
	Method to show O, A, C are colinear (e.g. show grad OA = grad AC or find the equation of AC and show that O lies on it).	M1		
	Accurate completion	A1	3	
(d)	Grad of $T_A=-\frac{3}{4}$	B1F		Or find $\frac{dy}{dx}=\frac{6-2x}{2y-8}\Rightarrow y'=-\frac{3}{4}$
	$T_A:\ y-\frac{4}{5}=-\frac{3}{4}\left(x-\frac{3}{5}\right)$	M1A1F		
	$15x+20y=25$	A1F	4	OE a, b, c integers
	$3x+4y=5$			
	Total		**10**	

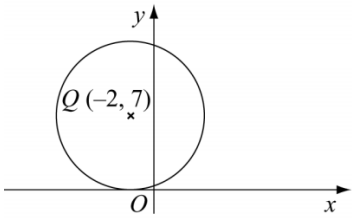
A circle has the equation

$$x^2+y^2+4x-14y+4=0.$$

(a) Find the radius of the circle and the coordinates of its centre. *(5 marks)*

(b) Sketch the circle. *(2 marks)*

(c) Find the length of a tangent from the point $P(6, 8)$ to the circle. *(4 marks)*

Q	Solution	Marks	Total	Comments
6 (a)	$x^2 + y^2 + 4x - 14y + 4 = 0$	M1		attempt to complete squares
	$(x+2)^2 + (y-7)^2 = 49$	A1 A1		for $(x+2)^2$ for $(y-7)^2$
	Radius 7 or $\sqrt{49}$	A1		(CAO)
	Centre (–2, 7)	A1F	5	
(b)		B1F B1F	2	Centre in 2nd quadrant Touching Ox
(c)	$PQ^2 = 8^2 + 1^2 (= 65)$	M1A1F		
	$PR^2 = PQ^2 - QR^2$	M1		
	$PR^2 = 65 - 49 = 16 \Rightarrow PR = 4$	A1F	4	
	Total		**11**	

A circle has equation

$$x^2 + y^2 - 4x + 4y - 12 = 0.$$

(a) Find:

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. *(5 marks)*

(b) Find the coordinates of the **two** points where the circle crosses the x-axis. *(3 marks)*

(c) Find the equation of the tangent to the circle at the point $(4, 2)$. *(4 marks)*

Q	Solution	Marks	Total	Comments
2 (a)(i)	Centre $(2\ ,\ -2)$	B1		
(ii)	Complete the square	M1		Attempted
	$(x-2)^2+(y+2)^2=20$	A1		LHS correct
		A1		RHS correct
	$\therefore r^2=20$			
	$r=\sqrt{20}$ or (AWRT 4.47)	A1√	5	(on their RHS > 0)
(b)	Crosses x-axis when $y=0$	M1		For use of $y=0$
	$\therefore x^2-4x-12=0$	m1		For solving quadratic by any correct method attempted
	$(x-6)(x+2)=0$			
	$x=6$ or $x=-2$			
	$\therefore$ crosses x-axis at the points $(6\ ,\ 0)$ & $(-2\ ,\ 0)$	A1	3	Accept $x=6$ *and* $x=-2$ if $y=0$ used
(c)	Slope of radius $=\dfrac{2--2}{4-2}=\dfrac{4}{2}=2$	B1√		On their centre
	Use $m_1m_2=-1$ for perpendicular lines			
	$\therefore$ slope of tangent $=-\dfrac{1}{2}$	B1√		On their slope of radius
	Equation of tangent is			If $m_1m_2=-1$ used then:
	$y-2=-\dfrac{1}{2}(x-4)$	M1		use of $y-y_1=m(x-x_1)$ or any other correct method
	$2y-4=-x+4$			
	$x+2y-8=0$	A1√	4	Accept any simplified form (on their value of m)
	Total		**12**	