Calculus

The function f is defined for $x \ge 0$ by

$$f(x) = x^{\frac{1}{2}} + 2$$
.

(a) (i) Find
$$f'(x)$$
. (2 marks)

(ii) Hence find the gradient of the curve y = f(x) at the point for which x = 4. (1 mark)

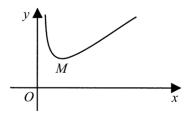
(b) (i) Find
$$\int f(x) dx$$
. (3 marks)

(ii) Hence show that
$$\int_0^4 f(x) dx = \frac{40}{3}$$
. (2 marks)

(c) Show that
$$f^{-1}(x) = (x-2)^2$$
. (2 marks)

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mathbf{f'}(x) = \frac{1}{2}x^{-\frac{1}{2}}$	M1A1	2	M1 if coefficient or index correct
(ii)	Gradient at $x = 4$ is $\frac{1}{4}$	A1F	1	ft wrong coeff
(b)(i)	$\int f(x)dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$	M1A1		M1 for $kx^{\frac{3}{2}}$
	$\dots + 2x (+c)$	B1	3	
(ii)	Substituting $x = 4$	M1		In c's integral (not $f(x)$ or $f'(x)$)
	$\int_0^4 \mathbf{f}(x) \mathrm{d}x = \frac{40}{3}$	A1	2	Convincingly found (AG)
(c)	$y = x^{\frac{1}{2}} + 2 \Rightarrow x^{\frac{1}{2}} = y - 2$	M1		OE
	$ \Rightarrow x = (y-2)^2$, hence result	A1	2	Convincingly shown (AG)

The curve with equation $y = 2x + \frac{27}{x^2} - 7$ is defined for x > 0, and is sketched below.



(a) (i) Find
$$\frac{dy}{dx}$$
. (3 marks)

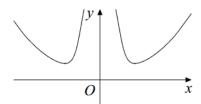
(ii) The curve has a minimum point M. Find the x-coordinate of M. (3 marks)

(b) (i) Find
$$\int \left(2x + \frac{27}{x^2} - 7\right) dx$$
. (3 marks)

(ii) Hence determine the area of the region bounded by the curve, the lines x = 1, x = 2 and the x-axis. (2 marks)

Question Number and part	Solution	Marks	Total marks	Comments
6(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \frac{54}{x^3}$	M1 A1 A1	3	Clearly attempting to differentiate One term correct but NOT $2 + f(x) - 7$ All correct (withhold if +c in answer)
(ii)	Putting "their" $\frac{dy}{dx} = 0$ $x^3 = 27$	M1		
	$x^3 = 27$	m1		Attempt at $x^3 = \dots$ or $x = \dots$ etc
	x = 3	A 1	3	
	$x = 3 x^2 - \frac{27}{x} - 7x (+c)$	M1 A1 A1	3	Clearly attempting to integrate Two terms correct All correct (ignore omission of $+c$)
(ii)	[4-13.5-14]-[1-27-7] = 9.5	M1		Evaluating limits at 1 and 2
	= 9.5	A1	2	
	Total		11	

A curve has equation $y = x^2 + \frac{81}{x^2}$. Its graph is sketched below.



- (a) (i) Find $\frac{dy}{dx}$. (3 marks)
 - (ii) Show that the stationary points of the curve occur when $x^4 = 81$. (2 marks)
 - (iii) Hence find the x-coordinates of the stationary points. (2 marks)
 - (iv) Find the value of the y-coordinate at each stationary point. (1 mark)

(b) (i) Find
$$\int \left(x^2 + \frac{81}{x^2}\right) dx$$
. (3 marks)

(ii) Hence find the area of the region bounded by the curve, the lines x = 1, x = 3 and the x-axis. (2 marks)

7(a)(i) $\frac{dy}{dx} = 2x$ $-\frac{162}{x^3}$ M1 A1 3 Power x^{-3} Putting candidate's $\frac{dy}{dx} = 0$ $\Rightarrow x^4 = 81 \text{ ag}$ A1 2 M1 only for verification $x = \pm 3$ (iii) $x^2 = 9$ or $x = \sqrt[4]{81}$ $x = \pm 3$ M1 Or $x = 3$ as only value given (iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 M1 No need to show both equal 18 B0 if 2 different y values given $x = \frac{x^3}{3}$ term					
(ii) $2x - \frac{162}{x^3} = 0$ $\Rightarrow x^4 = 81$ ag	7(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$	В1		
(ii) $2x - \frac{162}{x^3} = 0$ $\Rightarrow x^4 = 81$ ag M1 Putting candidate's $\frac{dy}{dx} = 0$ M1 only for verification $x = \pm 3$ (iii) $x^2 = 9$ or $x = \sqrt[4]{81}$ M1 Or $x = \pm 3$ Or $x = 3$ as only value given (iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{3}$ term x		$-\frac{162}{x^3}$			Power x^{-3}
$\Rightarrow x^4 = 81 \text{ ag} \qquad A1 \qquad 2 \qquad \text{M1 only for verification } x = \pm 3$ (iii) $x^2 = 9 \qquad \text{or } x = \sqrt[4]{81} \qquad \text{M1} \qquad 2$ Or $x = 3$ as only value given (iv) $y = 18 \qquad \qquad B1 \qquad 1 \qquad \text{No need to show both equal } 18$ (b)(i) $\frac{x^3}{3} = -\frac{81}{x} \qquad (+C)$ $B1 \qquad \qquad B1 \qquad \qquad x^3 \qquad \text{term} \qquad x^{-1} \text{power}$			A1	3	
(iii) $x^2 = 9$ or $x = \sqrt[4]{81}$ M1 Or $x = 3$ as only value given (iv) $y = 18$ B1 1 No need to show both equal 18 (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{3}$ term x^{-1} power	(ii)	$2x - \frac{162}{x^3} = 0$	M1		Putting candidate's $\frac{dy}{dx} = 0$
(iii) $x^2 = 9$ or $x = \sqrt[4]{81}$ M1 Or $x = 3$ as only value given (iv) $y = 18$ B1 1 No need to show both equal 18 (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{3}$ term x^{-1} power		$\Rightarrow x^4 = 81$ ag	A1	2	M1 only for verification $x = \pm 3$
(iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{3}$ term $$					
(iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{3}$ term $$	()	2			0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
(iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 x^3 term x^{-1} power	(111)	$x^2 = 9$ or $x = \sqrt[4]{81}$	M1		Or $x = 3$ as only value given
(iv) $y = 18$ B1 1 No need to show both equal 18 B0 if 2 different y values given (b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 $\frac{x^3}{M1}$ term $\frac{x^3}{x^{-1}}$ power		$x = \pm 3$	A1	2	
(b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 B0 if 2 different y values given $\frac{x^3}{x^{-1}}$ term $\frac{x^{-1}}{x}$ power					
(b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 B0 if 2 different y values given $\frac{x^3}{x^{-1}}$ term $\frac{x^{-1}}{x}$ power	(iv)	v = 18	B1	1	No need to show both equal 18
(b)(i) $\frac{x^3}{3} - \frac{81}{x}$ (+C) B1 x^3 term x^{-1} power	(11)			•	
$\frac{x}{3} - \frac{3}{x}$ (+C) M1 x^{-1} power					Bo if 2 differency values given
$\frac{1}{3}$ $\frac{1}{x}$ $\frac{1}{x}$ $\frac{1}{x}$ $\frac{1}{x}$ power	(b)(i)	x^3 81	B1		
		$\frac{1}{3}$ $-\frac{1}{r}$ $(+C)$	M1		x^{-1} power
A1 3 correct second term		3 4	A1	3	correct second term
(ii) For our Ft. 7 M1 Compet was of limits 1 and 2 substituted	(;;)	[MI		Compatives of limits 1 and 2 substituted
(ii) $\left\lceil \frac{27}{3} - \frac{81}{3} \right\rceil - \left\lceil \frac{1}{3} - 81 \right\rceil$ M1 Correct use of limits 1 and 3 substituted into answer for part (b)(i)	(11)	$\left \frac{27}{2} - \frac{81}{81} \right - \left \frac{1}{2} - 81 \right $	IVII		
					into answer for part (0)(1)
$62\frac{2}{3}$ Al 2 Accept 62.7 or better, condone 62.66 etc		$62\frac{2}{3}$	A1	2	Accept 62.7 or better, condone 62.66 etc
Total 13		Total		13	

The curve C has the equation

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, x > 0.$$

(a) Find the coordinates of the points where
$$C$$
 crosses the x -axis. (4)

(b) Find the exact coordinates of the stationary point of
$$C$$
. (5)

(d) Sketch the curve
$$C$$
. (2)

(a)
$$3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 0$$
, $3x^{\frac{1}{2}} - x - 2 = 0$ M1
 $x - 3x^{\frac{1}{2}} + 2 = 0$, $(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2) = 0$ M1
 $x^{\frac{1}{2}} = 1, 2$ A1
 $x = 1, 4 : (1, 0), (4, 0)$ A1

(b)
$$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$$
 M1 A1

for minimum,
$$-\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}} = 0$$
 M1
$$-\frac{1}{2}x^{-\frac{3}{2}}(x-2) = 0$$

$$x = 2, y = 3 - \sqrt{2} - \frac{2}{\sqrt{2}} \therefore (2, 3 - 2\sqrt{2})$$
 A2

(c)
$$\frac{d^2 y}{dx^2} = \frac{1}{4} x^{-\frac{3}{2}} - \frac{3}{2} x^{-\frac{5}{2}}$$
 M1

when
$$x = 2$$
, $\frac{d^2 y}{dx^2} = \frac{1}{8\sqrt{2}} - \frac{3}{8\sqrt{2}} = -\frac{1}{4\sqrt{2}}$, $\frac{d^2 y}{dx^2} < 0$: maximum A1

