Binomial Expansion

(a) Write down the first four terms in ascending powers of x in the expansion of

$$(1+x)^8,$$

simplifying your coefficients as much as possible. *(2 marks)*

(b) Find the coefficient of x^3 in the expansion of $(3-2x)(1+x)^8$. *(2 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
1(a)	$1+8x+28x^2+56x^3$ generous	M1 A1	 2	good attempt at coefficients or Pascal's triangle to 1 8 … must be simplified
(b)	$28\times -2+3\times 56$ " their 28 and 56" $=112$	M1 A1	 2	strict but condone + condone $112x^3$ even if not selected
	Total		**4**	

Expand $(3-2x)^4$ in ascending powers of x and simplify each coefficient. **(4)**

$= 3^4 + 4(3^3)(-2x) + 6(3^2)(-2x)^2 + 4(3)(-2x)^3 + (-2x)^4$ M1 A1
$= 81 - 216x + 216x^2 - 96x^3 + 16x^4$ B1 A1 **(4)**

(a) Expand $(1+3x)^8$ in ascending powers of x up to and including the term in x^3. You should simplify each coefficient in your expansion. **(4)**

(b) Use your series, together with a suitable value of x which you should state, to estimate the value of $(1.003)^8$, giving your answer to 8 significant figures. **(3)**

(a) $= 1 + 8(3x) + \binom{8}{2}(3x)^2 + \binom{8}{3}(3x)^3 + \ldots$ M1 A1
$= 1 + 24x + 252x^2 + 1512x^3 + \ldots$ M1 A1

(b) $x = 0.001$ B1
$(1.003)^8 \approx 1 + 0.024 + 0.000\,252 + 0.000\,001\,512$ M1
$= 1.024\,253\,5$ (8sf) A1 **(7)**

The coefficient of x^2 in the binomial expansion of $(1 + kx)^7$, where k is a positive constant, is 525.

(a) Find the value of k. **(3)**

Using this value of k,

(b) show that the coefficient of x^3 in the expansion is 4375, **(2)**

(c) find the first three terms in the expansion in ascending powers of x of

$$(2 - x)(1 + kx)^7.$$ **(3)**

(a) $(1 + kx)^7 = \ldots + \binom{7}{2}(kx)^2 + \ldots$ B1

$\therefore \frac{7\times 6}{2} \times k^2 = 525$

$k^2 = \frac{525}{21} = 25$ M1

$k > 0 \ \therefore k = 5$ A1

(b) $(1 + 5x)^7 = \ldots + \binom{7}{3}(5x)^3 + \ldots$

$\therefore$ coeff. of $x^3 = \frac{7\times 6\times 5}{3\times 2} \times 125 = 4375$ M1 A1

(c) $(1 + 5x)^7 = 1 + 35x + 525x^2 + \ldots$ B1

$(2 - x)(1 + 5x)^7 = (2 - x)(1 + 35x + 525x^2 + \ldots)$

$= 2 + 70x + 1050x^2 - x - 35x^2 + \ldots$ M1

$= 2 + 69x + 1015x^2 + \ldots$ A1 **(8)**