

Edexcel GCSE

Mathematics (Linear) – 1MA0

CIRCLE THEOREMS

Materials required for examination

Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Items included with question papers

Nil



Instructions

Use black ink or ball-point pen.

Fill in the boxes at the top of this page with your name, centre number and candidate number.

Answer all questions.

Answer the questions in the spaces provided – there may be more space than you need.

Calculators may be used.

Information

The marks for each question are shown in brackets – use this as a guide as to how much time to spend on **each** question.

Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed – you should take particular care on these questions with your spelling, punctuation and grammar, as well as the clarity of expression.

Advice

Read each question carefully before you start to answer it.

Keep an eye on the time.

Try to answer every question.

Check your answers if you have time at the end.

1.

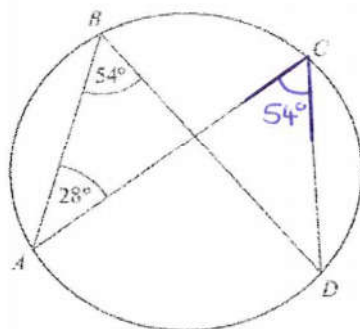


Diagram NOT
accurately drawn

A, B, C and D are points on the circumference of a circle.

Angle $ABD = 54^\circ$.

Angle $BAC = 28^\circ$.

(i) Find the size of angle ACD .

54 °

(ii) Give a reason for your answer.

Angles in the same segment are equal.

(3 marks)

2.

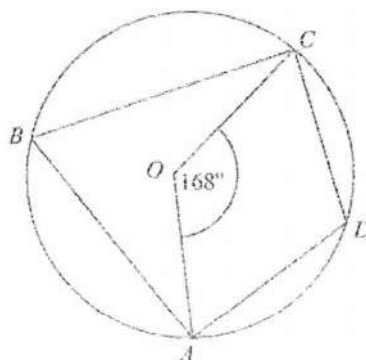


Diagram NOT
accurately drawn

A, B, C and D are points on the circumference of a circle, centre O .

Angle $AOC = 168^\circ$

Work out the size of angle ADC .

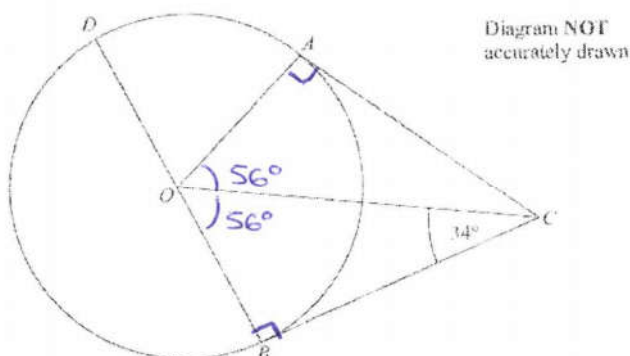
You must give reasons for your working.

$$\angle ABC = \frac{1}{2}(168) = 84^\circ \quad \text{Angles at the centre are twice angles at the circumference.}$$

$$\begin{aligned} \angle ADC &= 180 - \angle ABC \\ &= 180 - 84 \\ &= 96^\circ \end{aligned} \quad \text{Opposite angles in a cyclic quadrilateral have a sum of } 180^\circ.$$

(4 marks)

3.



A, B and D are points on the circumference of a circle, centre O .
 BOD is a diameter of the circle.
 BC and AC are tangents to the circle.
 Angle $OCB = 34^\circ$.

Work out the size of angle DOA .

$\angle CBO = 90^\circ$ A radius and tangent meet at 90° on the circumference of a circle.

$\angle BOC = 180 - (34 + 90)$ Angles in a triangle have a sum of 180° .
 $= 180 - 124$
 $= 56^\circ$

$\angle BOC = \angle COA$ as $\triangle BOC$ and $\triangle AOC$ are congruent

$\angle DOA = 180 - (\angle BOC + \angle COA)$ Angles on a straight line have a sum of 180°
 $= 180 - (56 + 56) = 68^\circ$ **68** (4 marks)

4.

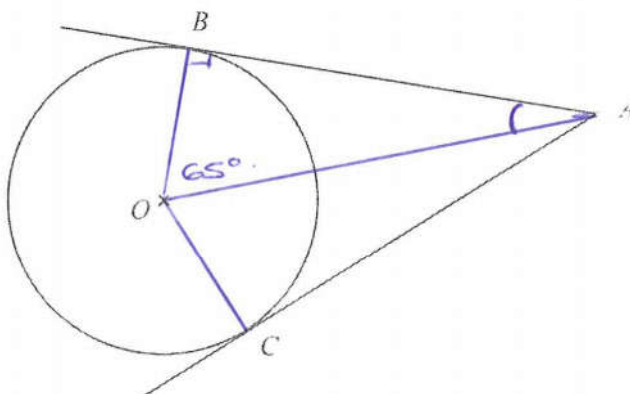


Diagram NOT accurately drawn

B and C are points on a circle, centre O .
 AB and AC are tangents to the circle.
 Angle $BOC = 130^\circ$.

Work out the size of angle BAO .

$\angle BOA = \frac{1}{2} \angle BOC = \frac{1}{2} (130) = 65^\circ$

$\angle OBA = 90^\circ$ Tangent and radius meeting at circumference

$\angle BAO = 180 - (65 + 90)$
 $= 180 - 155$
 $= 25^\circ$

(4 marks)

5.

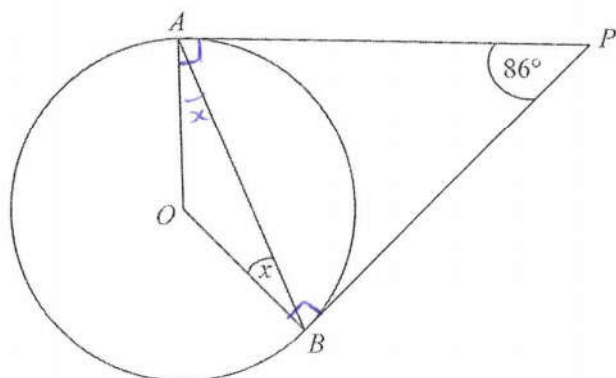


Diagram NOT accurately drawn

A and B are points on the circumference of a circle, centre O .
 PA and PB are tangents to the circle.
 Angle APB is 86° .

Work out the size of the angle marked x .

$$\angle PAO = \angle PBO = 90^\circ \text{ (Tangent and radius meeting)}$$

$$\angle BOA = 360 - (90 + 90 + 86) = 94^\circ$$

$\triangle BOA$ is isosceles as $OB = OA$ (both radii)

$$180 = 94 + 2x \Rightarrow 2x = 86$$

$$x = 43^\circ$$

..... 43

(3 marks)

6.

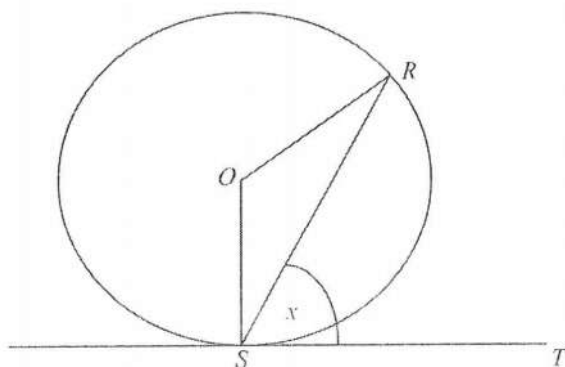


Diagram NOT accurately drawn

R and S are two points on a circle, centre O .
 TS is a tangent to the circle.
 Angle $RST = x$.

Prove that angle $ROS = 2x$.

You must give reasons for each stage of your working.

$$\angle OST = 90^\circ \text{ Tangent and Radius meeting at a circumference.}$$

$$\angle RSO = 90 - x$$

$$\angle SRO = \angle RSO = 90 - x \text{ as } \triangle RSO \text{ is isosceles, } RO = SO \text{ (both radii)}$$

$$\angle ROS = 180 - (90 - x) - (90 - x) \text{ Sum of angles in a triangle is } 180^\circ$$

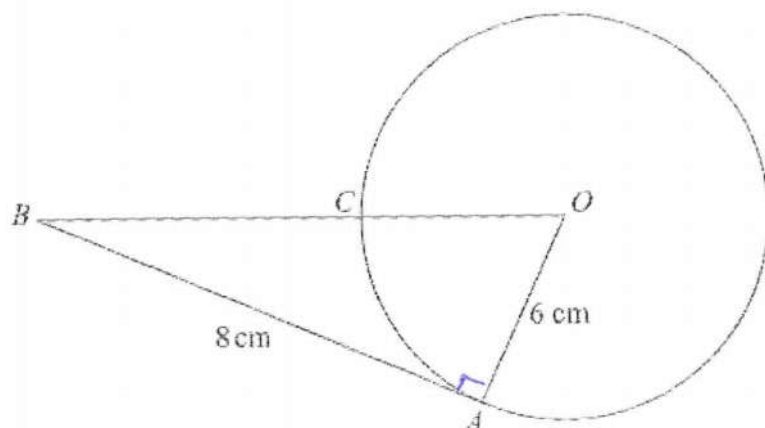
$$= 180 - 90 + x - 90 + x$$

$$= 2x$$

(4 marks)

7.

Diagram NOT
accurately drawn



In the diagram, O is the centre of the circle.
 A and C are points on the circumference of the circle.
 BCO is a straight line.
 BA is a tangent to the circle.

$AB = 8$ cm.

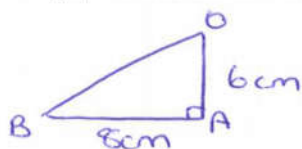
$OA = 6$ cm.

(a) Explain why angle OAB is a right angle.

A tangent and radius meet at 90° on the circumference of a
circle.

(1)

(b) Work out the length of BC .



Using Pythagoras

$$\begin{aligned} OB^2 &= 6^2 + 8^2 \\ &= 100 \\ OB &= \sqrt{100} \\ &= 10 \text{ cm} \end{aligned}$$

$OC = 6$ cm (radius of the circle).

$$\begin{aligned} \text{so } BC &= 10 - 6 \\ &= \underline{\underline{4 \text{ cm}}} \end{aligned}$$

4.....cm
(3)

(4 marks)

8.

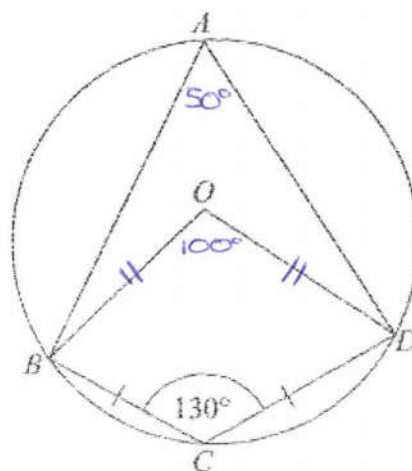


Diagram NOT
accurately drawn

A, B, C and D are points on a circle, centre O .
 $BC = CD$.
Angle $BCD = 130^\circ$.

- (a) Write down the size of angle BAD .
Give a reason for your answer.

$$\angle BAD = 180 - 130^\circ$$

$$= 50^\circ$$

Opposite angles in a cyclic quadrilateral have
a sum of 180° .

..... 50°
(2)

- (b) Work out the size of angle ODC .
Give reasons for your answer.

$$\angle BOD = 2 \times \angle BAD$$

$$= 2 \times 50$$

$$= 100^\circ$$

Angle at the centre twice that
at the circumference.

$OBCD$ is a kite (pairs of adjacent equal sides).
therefore $\angle OBC = \angle ODC$. ($= x$)

so $360 - 100 - 130 = 2x$ Angles in a quadrilateral have a
sum of 360°

$$2x = 130$$

$$x = 65^\circ$$

..... 65°
(4)

(6 marks)

9.

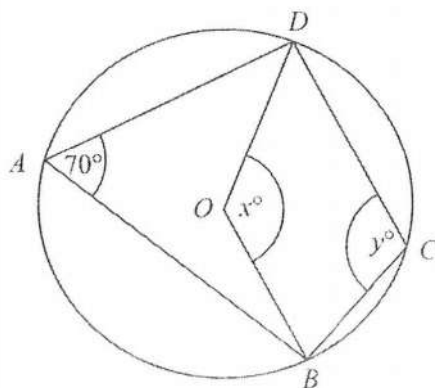


Diagram NOT accurately drawn

In the diagram, A , B , C and D are points on the circumference of a circle, centre O .
 Angle $BAD = 70^\circ$.
 Angle $BOD = x^\circ$.
 Angle $BCD = y^\circ$.

- (a) (i) Work out the value of x .

$$x = 140^\circ$$

- (ii) Give a reason for your answer.

Angle at the centre is twice the angle at the circumference

(2)

- (b) (i) Work out the value of y .

$$y = 110^\circ$$

- (ii) Give a reason for your answer.

Opposite angles in a cyclic quadrilateral have a sum of 180°

(2)

(4 marks)

10.

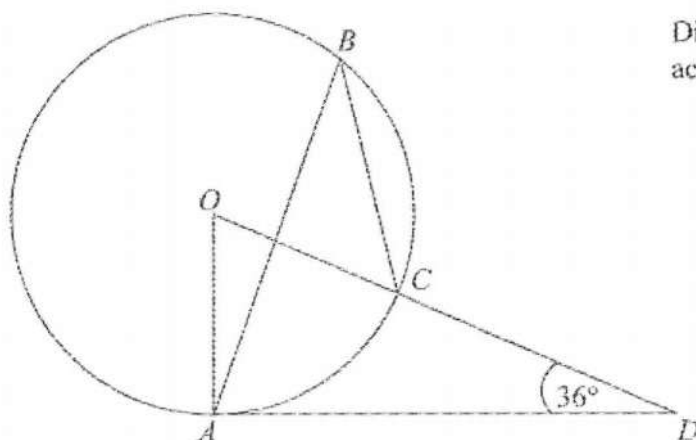


Diagram NOT
accurately drawn

The diagram shows a circle centre O .
 A , B and C are points on the circumference.

DCO is a straight line.

DA is a tangent to the circle.

Angle $ADO = 36^\circ$

(a) Work out the size of angle AOD .

$\angle DAO = 90^\circ$ (Radius and tangent meet at right angles
at the circumference of a circle)

$\angle AOD = 180 - 90 - 36$ (Sum of angles in a triangle is 180°)
 $= 54^\circ$

..... 54
(2)

(b) (i) Work out the size of angle ABC .

$\angle ABC = \frac{1}{2} \angle AOC$
 $= \frac{1}{2} \angle AOD$
 $= \frac{1}{2} (54)$
 $= 27^\circ$

..... 27
(2)

(ii) Give a reason for your answer.

..... The angle at the centre is twice the angle at the circumference,

..... So $\angle AOC = 2 \angle ABC$

.....
(3)
(4 marks)

11.

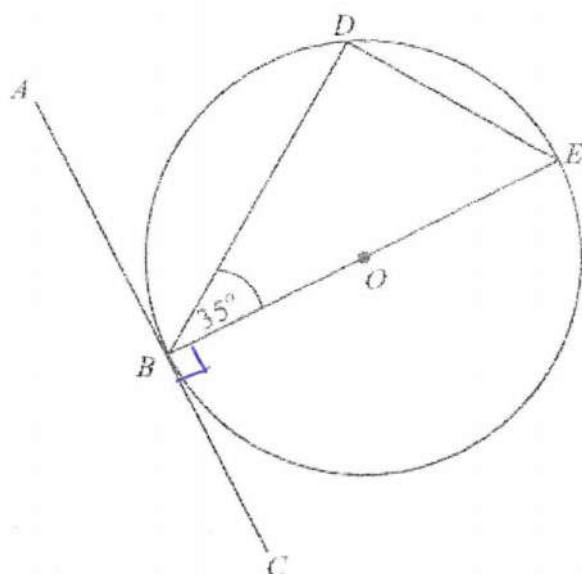


Diagram NOT
accurately drawn

B , D and E are points on a circle centre O .
 ABC is a tangent to the circle.
 BE is a diameter of the circle.
 Angle $DBE = 35^\circ$.

(a) Find the size of angle ABD .

Give a reason for your answer.

$\angle CBE = 90^\circ$ Tangent and Radius meet at 90° at
 The circumference of a circle

$\angle ABD = 180 - (90 + 35)$ Angles on a straight line have
 $= 180 - 125$ a sum of 180°
 $= 55^\circ$

..... 55
 (2)

(b) Find the size of angle DEB .

Give a reason for your answer.

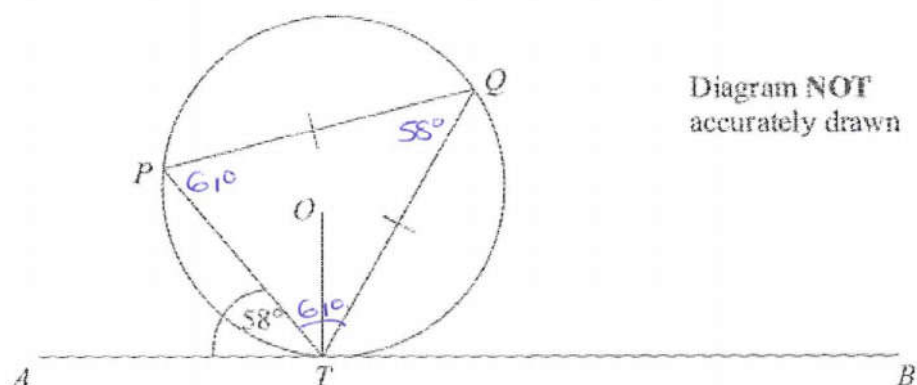
$\angle BDE = 90^\circ$ Angle opposite the diameter in a semi-circle.

~~40~~
 $\angle DEB = 180 - (90 + 35)$ Angles in a triangle have a
 $= 180 - 125$ sum of 180°
 $= 55^\circ$

..... 55
 (2)

(4 marks)

12.



P , Q and T are points on the circumference of a circle, centre O .
The line ATB is the tangent at T to the circle.

$PQ = TQ$.
Angle $ATP = 58^\circ$.

Calculate the size of angle OTQ .
Give a reason for each stage in your working.

$$\angle TQR = 58^\circ \quad \text{Alternate Segment Theorem}$$

$$\angle QPT = \angle PQT \quad \text{Isosceles triangle}$$

$$\angle QPT = \frac{180 - 58}{2} = 61^\circ$$

$$\angle OTQ = 61 - \angle OTP$$

$$\angle OTP = 90^\circ - 58^\circ \quad \text{Tangent and radius meet at } 90^\circ$$

$$= 32^\circ$$

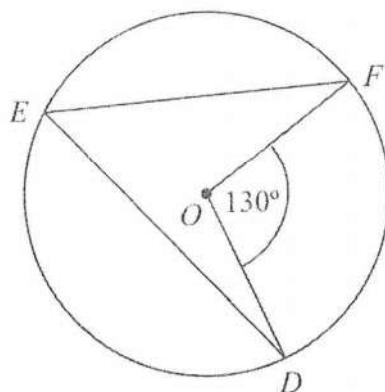
$$\angle OTQ = 61 - 32^\circ$$

$$= 29^\circ$$

29°

(4 marks)

13. (a)



D, E and F are points on the circumference of a circle, centre O .
Angle $DOF = 130^\circ$.

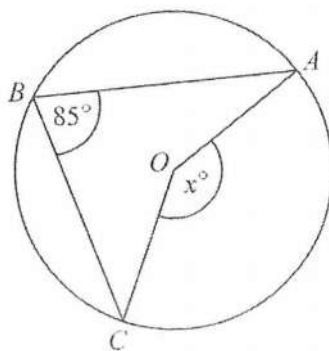
- (i) Work out the size of angle DEF .

..... 65 $^\circ$

- (ii) Give a reason for your answer.

..... Angle at the circumference is half the angle at the
..... centre
.....

(2)



(b)

In the diagram, A, B and C are points on the circumference of a circle, centre O .

Angle $ABC = 85^\circ$.

- (i) Work out the size of the angle marked x° .

..... 170 $^\circ$

- (ii) Give a reason for your answer.

..... Angle at the centre is twice the angle at the circumference
.....

(2)

(4 marks)

*14.

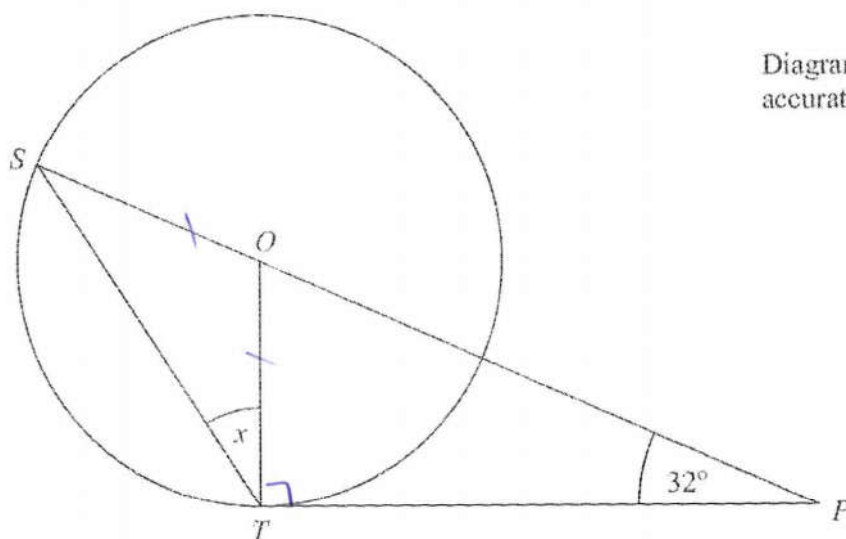


Diagram NOT
accurately drawn

S and T are points on the circumference of a circle, centre O .
 PT is a tangent to the circle.
 SOP is a straight line.

Angle $OPT = 32^\circ$.

Work out the size of the angle marked x .
 Give reasons for your answer.

$\angle PTO = 90^\circ$ Tangent and radius meet at 90° on the circumference of a circle.

$\angle POT = 180 - (90 + 32)$ Angles in a triangle have a sum of 180° .
 $= 180 - 122$
 $= 58^\circ$

$\angle TOS = 180 - 58$ Angles on a straight line have a sum of 180° .
 $= 122^\circ$

$\triangle OTS$ is isosceles as $OS = OT$ (both radii) so $\angle OST = \angle STO$

$$122 + 2x = 180$$

$$2x = 58$$

$$\underline{x = 29^\circ}$$

.....29.....°

(Total 5 marks)