

## Further Pure 1 Essential skills 1

Student Name:	Target:
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1	The roots of the equation $x^2 - 3x + 6 = 0$ are $\alpha$ and $\beta$ Find the value of $\alpha^2 + \beta^2$	$-3$
2	Using the roots above, find a quadratic equation with roots $\alpha^2$ and $\beta^2$	$x^2 + 3x + 36 = 0$
3	Solve $x^2 - 6x + 25 = 0$ giving each root in the form $a + bi$	$3 \pm 4i$
4	Find the complex number $z$ such that $5z + 2z^* = 35 + 6i$	$5 + 2i$
5	Solve the inequality $\frac{1}{x+5} > 2$	$-5 < x < -4.5$
6	Work out $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$	$\begin{bmatrix} -2 & 11 \\ -10 & 22 \end{bmatrix}$
7	Describe the transformation associated with the matrix: $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	stretch in x direction by SF 3
8	Find the matrix that generates a rotation of $30^\circ$ anticlockwise	$\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$
9	Find the general solution, in radians, of the equation $\sin 2x = -\frac{\sqrt{3}}{2}$	$x = \pi n - \frac{\pi}{6}$ or $x = \pi n - \frac{\pi}{3}$
10	Find the general solution, in degrees, of $2 \cos^2 x = \cos x$	$x = 360n$ and $x = 360n \pm 60^\circ$
11	Write $y = ax^n$ in the form $Y = mX + c$ Indicate clearly each of Y, X, m and c	$\ln y = n \ln x + \ln a$ $Y = mX + C$
12	A curve has equation $y = x^3$ The points P and Q lie on the curve and have x-coordinates 3 and $3+h$ . Find the gradient of the curve at point P.	$\frac{dy}{dx} = 27 + 9h + h^2$ As $h \rightarrow 0$ $\frac{dy}{dx} = 27$
13	Determine whether the following integral exists and, if so, find its value. $\int_1^\infty x^{-2} dx$	As $p \rightarrow \infty$ $\int dx \rightarrow 1$
14	Find $\sum_{r=1}^n 4r^2 + 1$	$\frac{2}{3}n(2n^2 + 3n + \frac{5}{2})$
15	Therefore find $\sum_{r=1}^{20} 4r^2 + 1$	11500
16	Use Newton Raphson method to find $x_2$ for $x^3 - 6x + 12 = 0$ given $x_1 = 2$	$2/3$
17	Show that the equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2. Use linear interpolation to find this root.	$\alpha = 1.25$
18	Find the equations of the asymptotes of $y = \frac{5-2x}{x+1}$	$x = -1$ $y = -2$
19	Use Euler's step-by-step method, with a step length of 0.1 to estimate the value of $y$ when $x = 0.3$ , given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = \frac{1}{x^2 + 4}$ giving your answer to 3 significant figures.	1.07
20	Find the possible values of $m$ such that the line $y = mx - 7$ is a tangent to the hyperbola with equation $x^2 - 2y^2 = 14$	$\pm 2$

## Further Pure 1 Essential skills 2

Student Name: _____		Target: _____
1	The roots of the equation $x^2 - 5x + 4 = 0$ are $\alpha$ and $\beta$ Find the value of $\frac{3}{\alpha} + \frac{3}{\beta}$	$\frac{15}{4}$
2	Using the roots above, find a quadratic equation with roots $\frac{3}{\alpha}$ and $\frac{3}{\beta}$	$4x^2 - 15x + 9 = 0$
3	Solve $x^2 + 4x + 5 = 0$ giving each root in the form $a + bi$	$-2 + i, -2 - i$
4	Find the complex number $z$ such that $9z - 3z^* = 24 - 12i$	$z = 4 - i$
5	Solve the inequality $\frac{6x}{x-1} < 3$	$-1 < x < 1$
6	Work out $\begin{bmatrix} -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -5 & -1 \\ 0 & -5 \end{bmatrix}$	$\begin{bmatrix} 15 & -7 \\ -20 & -19 \end{bmatrix}$
7	Describe the transformation associated with the matrix: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	Reflect in $x$ axis.
8	Write down the transformation that has the matrix: $\begin{bmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{bmatrix}$	Clockwise Rotation about $(0,0)$ $53.13^\circ$
9	Find the general solution, in radians, of the equation $\cos 4x = 0.6$	$\frac{1}{2}\pi n \pm 0.232^\circ$
10	Find the general solution, in degrees, of $4\sin^2 2x = 3$	$90^\circ n \pm 30^\circ$
11	Write $y = ax^2 + \frac{b}{x}$ in the form $Y = mX + c$ Indicate clearly each of $Y, X, m$ and $c$	$xy = ax^3 + b$ $Y = mX + b$
12	A curve has equation $y = x^2 - 4x + 5$ The points $P$ and $Q$ lie on the curve and have $x$ -coordinates 2 and $2+h$ . Find the gradient of the curve at point $P$ .	$\frac{dy}{dx} = h$ As $h \rightarrow 0$ $\frac{dy}{dx} = 0$
13	Determine whether the following integral exists and, if so, find its value. $\int_1^\infty x^2 dx = \left[ \frac{x^3}{3} \right]_1^\infty = \frac{p^3}{3} - \frac{1}{3}$	As $p \rightarrow \infty$ $\int dx \rightarrow$ NO value
14	Find $\sum_{r=1}^n 6r^2 + 4r$	$n(n+1)(2n+3)$
15	Therefore find $\sum_{r=1}^{50} 6r^2 + 4r$	262650
16	Use Newton Raphson method to find $x_2$ for $x^4 - 2x + 5 = 0$ given $x_1 = -2$	$\frac{-43}{34} = -1\frac{9}{34}$
17	Show that the equation $x^5 - 4x - 50 = 0$ has a root between 2 and 3. Use linear interpolation to find this root.	$f(2) = -26$ $f(3) = 181$ $\alpha = 2.46$
18	Find the equations of the asymptotes of $y = \frac{3-4x}{2x-5}$	$x = 5/2$ $y = -2$
19	Use Euler's step-by-step method, with a step length of 0.2 to estimate the value of $y$ when $x = 1.6$ , given that $y = 4$ when $x = 1$ and $\frac{dy}{dx} = \frac{1}{3-x}$ giving your answer to 3 significant figures.	4.34
20	Describe the transformation that changes $\frac{x^2}{4} - y^2 = 1$ to $\frac{(x-1)^2}{4} - (y+1)^2 = 1$	TRANSLATE $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

change of sign root between 2 & 3.

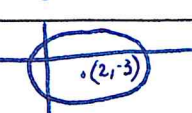


## Further Pure 1 Essential skills 3

Student Name:	Target:
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1	The roots of the equation $2x^2 - 6x + 5 = 0$ are $\alpha$ and $\beta$ Find the value of $\alpha^2 + \beta^2$	4
2	Using the roots above, find a quadratic equation with roots $\alpha^2 + 2$ and $\beta^2 + 2$	$4x^2 - 32x + 73 = 0$
3	Solve $x^2 - 12x + 40 = 0$ giving each root in the form $a + bi$ .	$6 \pm 2i$
4	Find the complex number $z$ such that $2z - 5z^* = 15 - 14i$	$-5 - 2i$
5	Solve the inequality $\frac{x-1}{x-4} \leq 2$	$x \geq 7, x < 4$
6	Work out $\begin{bmatrix} 3 & 2 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 4 & -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 17 & -5 & 6 \\ -21 & 8 & -6 \end{bmatrix}$
7	Describe the transformation associated with the matrix: $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$	Stretch SF 5 in x direction Stretch SF 2 in y direction
8	Find the matrix that generates a reflection in the line $y = \sqrt{3}x$	$\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$
9	Find the general solution, in radians, of the equation $\tan(x + 20^\circ) = \tan 60^\circ$	$x = \pi n + \frac{2\pi}{9}$
10	Find the general solution, in degrees, of $2\sin x \cos x - \cos x = 0$	$x = 360n \pm 90$ and $x = 360n + 360$ and $x = 360n + 150$
11	Write $y = ab^{-x}$ in the form $Y = mX + c$ Indicate clearly each of Y, X, m and c	$\ln y = (\ln b)(-x) + \ln a$ $Y = mX + C$
12	A curve has equation $y = x^2 - 3x + 7$ The points P and Q lie on the curve and have x-coordinates 3 and $3+h$ . Find the gradient of the curve at point P.	$\frac{dy}{dx} = 2x - 3$ at P $\frac{dy}{dx} = 3$
13	Determine whether the following integral exists and, if so, find its value. $\int_{-\infty}^{-2} \frac{1}{x^3} dx$ $\int_p^{-2} x^{-3} dx = \left[ \frac{x^{-2}}{-2} \right]_p^{-2} = \left[ \frac{1}{-2x^2} \right]_p^{-2} = \frac{-1}{8} + \frac{1}{2p^2}$	As $p \rightarrow \infty$ $\int dx \rightarrow -\frac{1}{8}$
14	Find $\sum_{r=1}^n 3r^3 + 2r$	$\frac{1}{4} n(n+1)(3n^2 + 3n + 4)$
15	Therefore find $\sum_{r=1}^{10} 3r^3 + 2r$	9185
16	Use Newton Raphson method to find $x_2$ for $x^3 - 2x + 3 = 0$ given $x_1 = -3$	-2.23
17	Show that the equation $2^x - 2x - 6 = 0$ has a root between 3 and 4. Use linear interpolation to find this root.	$f(3) = -4, f(4) = 2$ $\alpha = 11/3$ or $3.66$
18	Find the equations of the asymptotes of $y = \frac{2x+7}{x^2-3x+2}$	$x=2$ $y=0$ $x=1$
19	Use Euler's step-by-step method, with a step length of 0.5 to estimate the value of $y$ when $x=1$ , given that $y=1$ when $x=0$ and $\frac{dy}{dx} = \frac{1}{3-x}$	$y = 1.36$
20	Find the possible values of $m$ such that the line $y = mx + 10$ is a tangent to the ellipse with equation $4x^2 + y^2 = 20$	$m = \pm 4$

## Further Pure 1 Essential skills 4

Student Name:		Target:
1	The roots of the equation $3x^2 - 9x + 2 = 0$ are $\alpha$ and $\beta$ Find the value of $\alpha + \beta$ and $\alpha\beta$	$\alpha + \beta = 3$ $\alpha\beta = 2/3$
2	Using the roots above, find a quadratic equation with roots $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$	$8x^2 - 54x + 27 = 0$
3	Solve $5x^2 - 6x + 5 = 0$ giving each root in the form $a + bi$	$z = \frac{3 \pm 4i}{5}$
4	Find the complex number $z$ such that $z + 8z^* = 27 - 7i$	$z = 3 + i$
5	Solve the inequality $\frac{2x+3}{x-2} \leq 1$	$5 \leq x \leq 2$
6	Work out $\begin{bmatrix} -1 & 3 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} -1 & -1 & -3 & 3 \\ 2 & 3 & 1 & 5 \end{bmatrix}$	$\begin{bmatrix} 7 & 10 & 6 & 12 \\ -13 & -17 & -19 & -5 \end{bmatrix}$
7	Describe the transformation associated with the matrix: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	reflect in line $y = x$
8	Write down the transformation that has the matrix: $\begin{bmatrix} 0.342 & 0.940 \\ 0.940 & -0.342 \end{bmatrix}$	reflection line $y = \tan 35^\circ x$ or $y = 0.7x$
9	Find the general solution, in radians, of the equation $\sin(x + 40^\circ) = \sin 50^\circ$	$2\pi n + \frac{\pi}{8} \div 0R$ $2\pi n + \frac{7\pi}{8} \div 2\pi n + \frac{\pi}{2}$
10	Find the general solution, in degrees, of $\cos x + 1 = 2\sin^2 x$	$x = 360n \pm 60$ $x = 360n \pm 180$
11	Write $y = bx^a$ in the form $Y = mX + c$ Indicate clearly each of Y, X, m and c	$\ln y = a \ln x + \ln b$ $Y = mX + c$
12	A curve has equation $y = 2x^2 - 5x + 4$ The points P and Q lie on the curve and have x-coordinates 5 and $5+h$ . Find the gradient of the curve at point P.	$\frac{\partial y}{\partial x} = 15 + 2h$ so $\frac{dy}{dx} = 15$
13	Determine whether the following integral exists and, if so, find its value. $\int_0^4 \frac{1}{\sqrt{x}} dx$	As $p \rightarrow 0$ $\int dx \rightarrow 4$
14	Find $\sum_{r=1}^n 4r^3 + 2r$	$n(n+1)(n^2+n+1)$
15	Therefore find $\sum_{r=11}^{20} 4r^3 + 2r$	164610
16	Use Newton Raphson method to find $x_2$ for $x^4 - 5x^2 + 1 = 0$ given $x_1 = 3$	$\frac{197}{78} = 2.53$
17	Show that the equation $x^3 + 2x - 5 = 0$ has a root between 1 and 1.5 Use linear interpolation to find this root.	$f(1) = -2$ $f(1.5) = 1.375$ Change of sign $x = 1.296$
18	Find the equations of the asymptotes of $y = \frac{1+4x}{x^2-3x+6}$	$y = 0$
19	Use Euler's step-by-step method, with a step length of 0.01 to estimate the value of $y$ when $x = 1.02$ , given that $y = 4$ when $x = 1$ and $\frac{dy}{dx} = 2^x$	$y = 4.04$
20	Sketch the curve $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{25} = 1$ indicating where it crosses the co-ordinate axes- <i>the centre of the conic</i>	



## Further Pure 1 Essential skills 5

Student Name:	Target:
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1	The roots of the equation $3x^2 - 9x + 2 = 0$ are $\alpha$ and $\beta$ Find the value of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$	$\frac{23}{3}, 21$
2	Using the roots above, find a quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$	$6x^2 - 189x + 4 = 0$
3	Solve $x^2 + 4x + 9 = 0$ giving each root in the form $a + bi$	$-2 \pm \sqrt{5}i$
4	Find the complex number $z$ such that $\frac{2}{3}z + \frac{3}{5}z^* = 15 - 30i$	$\frac{225}{19} - 450i$
5	Solve the inequality $\frac{x+3}{2x-1} \geq 2$	$\frac{1}{2} < x \leq \frac{5}{3}$
6	Work out $\begin{bmatrix} -3 & -7 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} -3 & 4 & 6 & 11 \\ 4 & -3 & -9 & 5 \end{bmatrix}$	$\begin{bmatrix} -19 & 9 & 45 & -68 \\ 30 & -32 & -84 & 18 \end{bmatrix}$
7	Describe the transformation associated with the matrix: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	Rotate $90^\circ$ anticlockwise about
8	Find the matrix that generates a rotation of $120^\circ$ anticlockwise	$\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$
9	Find the general solution, in radians, of the equation $\cos(40^\circ - 2x) = \cos 10^\circ$	$x = -\pi n + \frac{1}{12}\pi$ $x = -\pi n - \frac{5}{36}\pi$
10	Find the general solution, in degrees, of $2\cos^2 x = 2 + \sin x$	$x = 180^\circ$ $x = 360n - 30^\circ$ and $x = 360n - 150^\circ$
11	Write $\frac{1}{y} = ax + bx^2$ in the form $Y = mX + c$ Indicate clearly each of Y, X, m and c	$\frac{1}{9x} = bx + a$ $Y = mX + c$
12	A curve has equation $y = x^3 - x^2$ The points P and Q lie on the curve and have x-coordinates 3 and $3+h$ . Find the gradient of the curve at point P.	$\frac{\delta y}{\delta x} = \frac{21h + 8h^2 + h^3}{h}$ As $h \rightarrow 0$ $\frac{dy}{dx} = 21$
13	Determine whether the following integral exists and, if so, find its value. $\int_0^8 x^{\frac{1}{3}} dx = \left[ \frac{3x^{2/3}}{2} \right]_0^8 = \frac{12}{2} - \frac{3 \cdot 0^{2/3}}{2}$	As $p \rightarrow 0$ $\int dx \rightarrow 6$
14	Find $\sum_{r=1}^n 2r^3 + 3r^2 + 1$	$\frac{1}{2}n(n^3 + 4n^2 + 4n + 3)$
15	Therefore find $\sum_{r=41}^{50} 2r^3 + 3r^2 + 1$	1968815
16	Use Newton Raphson method to find $x_2$ for $24x^3 + 36x^2 + 18x - 5 = 0$ given $x_1 = 0.2$	0.193
17	Show that the equation $\sin x + 3x - 5 = 0$ has a root between 1 and 2. Use linear interpolation to find this root. (Put calc in radians)	$f(1) = -1.1585$ $f(2) = 1.9093$ change of sign. $\alpha = 1.38$
18	Find the equations of the asymptotes of $y = \frac{2x-5}{x^2-4}$	$x = 2, x = -2$ $y = 0$
19	Use Euler's step-by-step method, with a step length of 0.25 to estimate the value of $y$ when $x = 2.5$ , given that $y = -1$ when $x = 2$ and $\frac{dy}{dx} = \sqrt{x^4 + 9}$	6.13
20	Find the possible values of $m$ such that the line $y = m(x+2)$ is a tangent to the parabola with equation $y^2 = x - 2$	$m = \pm \frac{1}{4}$